NAME __Key__

Math 12
Test 2
Spring 2011

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Suppose $f(x) = x^4 + 8x^3 + 18x^2 - 8$. Find all critical numbers, list the intervals of increase and decrease, and tell whether each critical number will result in a maximum, a minimum, or neither.

$$f'(x) = 4x^3 + 24x^2 + 36x$$
$$= 4x(x^2 + 6x + 9)$$
$$= 4x(x+3)^2$$

CN: $x = 0,\ x = -3$

f': $\ominus$ | $\ominus$ | $\oplus$ at -3, 0

min

dec on $(-\infty, -3) \cup (-3, 0)$

inc on $(0, \infty)$

$x = -3$ gives neither

$x = 0$ gives a min

2. For the following functions, find all horizontal and vertical asymptotes (remember that an asymptote is a LINE, not a number). If there are no asymptotes, say so.

(a) $f(x) = \sqrt{\dfrac{x^3 + 1}{4x^3}}$

VA: $x = 0$

HA: $y = 1/2$

(b) $f(x) = \dfrac{3x + 3}{x^2 - x - 2} = \dfrac{3(x+1)}{(x-2)(x+1)}$

VA: $x = 2$ (hole at $x = -1$)

HA: $y = 0$

(c) $f(x) = \dfrac{4x - 5}{2}$

VA: none

HA: none

3. a) Suppose the marginal cost of producing 10 units of a product is \$1, and the total cost to produce 10 units is \$15. Does the production of the 11th unit cause the average cost per unit produced to get bigger, smaller, or stay the same? Explain.

AC for 10 units is $\frac{15}{10} = \$1.50$ per unit
next unit only costs about \$1, so average cost will decrease
(MC < AC) AC $\approx \frac{16}{11}$ for 11 units, $\approx \$1.45$ per unit

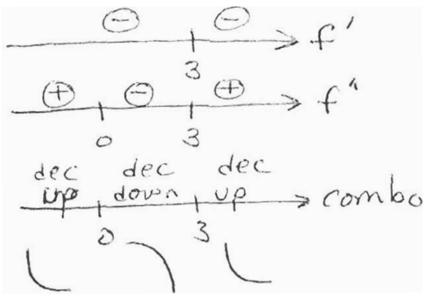
b) Give an example of a product whose demand function would, in general, be elastic. What does it mean (in words, not an equation) for demand to be elastic?

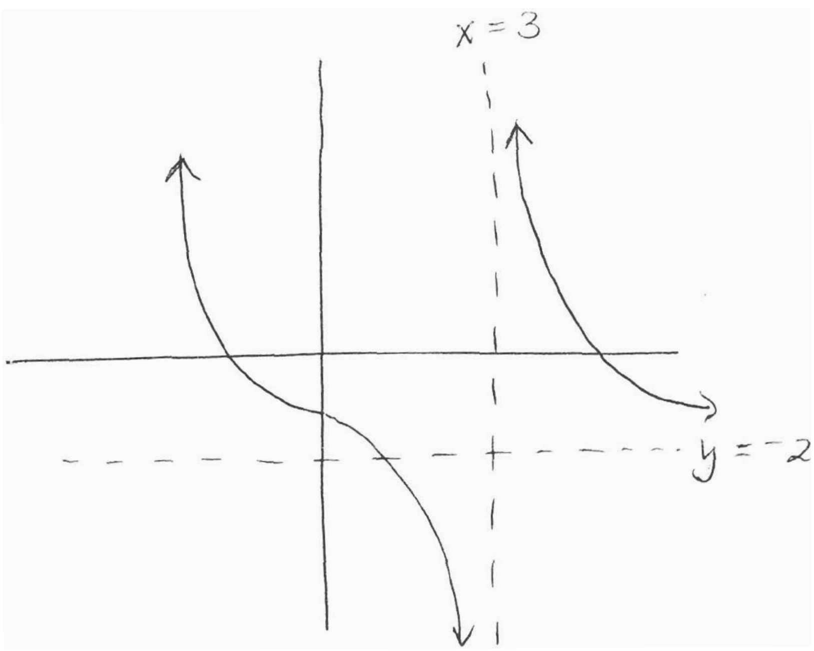
Elastic demand is for luxury products, like vacations, jewelry, designer clothes, etc.

Demand is elastic when demand decreases more than 1% for a 1% increase in price, ie, demand falls out of proportion to the price increase.

4. Sketch a nice BIG graph of a function with all the properties listed below. Make sure your graph is clearly labeled.

a) $f'(x) < 0$ for all values of x except $x = 3$
b) $f''(x) < 0$ for $0 < x < 3$, but $f''(x) \geq 0$ otherwise
c) $f(x)$ is undefined when $x = 3$ hole or asymp
d) $\lim_{x \to \infty} f(x) = -2$. asymp $y = -2$ on right side





5. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = \dfrac{3x+1}{\sqrt{1-4x}} = \dfrac{3x+1}{(1-4x)^{1/2}}$

$$f'(x) = \frac{(3)(1-4x)^{1/2} - (3x+1)\left(\frac{1}{2}\right)(1-4x)^{-1/2}(-4)}{1-4x}$$

(b) $f(x) = (x^2-3)^5(2x-1)^3$

$$f'(x) = 5(x^2-3)^4(2x)(2x-1)^3 + (x^2-3)^5(3)(2x-1)^2(2)$$

6. Find all the points where the line tangent to $x^2 + xy + y^2 = 3$ is horizontal.

$2x + (1)(y) + (x)(1)(y') + 2yy' = 0$

$2x + y + xy' + 2yy' = 0$

$y'(x+2y) = -2x - y$

$y' = \dfrac{-2x-y}{x+2y}$

Horizontal tangent means $y' = 0$, which occurs when $-2x - y = 0$,

$y = -2x$.

$x^2 + x(-2x) + (-2x)^2 = 3$

$x^2 - 2x^2 + 4x^2 = 3$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1, \ y = -2, 2.$

points:

$(1,-2)$ and $(-1,2)$

7. Find the absolute minimum and absolute maximum *points* of $f(x) = (x^2 - 4)^5$ on the interval $-3 \le x \le 2$.

$f'(x) = 5(x^2-4)^4(2x) = 0$

CN $x = 0,\ x = \pm 2$

$f(-3) = (9-4)^5 = 5^5$ $\quad (-3, 3125) \leftarrow$ abs max

$f(-2) = 0$ $\quad (-2, 0)$

$f(0) = (-4)^5$ $\quad (0, -1024) \leftarrow$ abs min

$f(2) = 0$ $\quad (2, 0)$

8. A commercial fruit grower must decide when to pick his pears. If he does it now, the pears will bring 32 cents per pound and each tree will yield 60 pounds of pears. Over the next 4 weeks, the yield per tree will increase 9 pounds per week, but the price per pound will decrease 3 cents per week. In order to maximize revenue, when should he pick ~~to~~ the pears? (Hint: It might be easier to work in pennies rather than dollars).

x = # weeks until picking.

Revenue = (price)(quantity)

$R = (32 - 3x)(60 + 9x)$

$R' = -3(60+9x) + (32-3x)(9)$

$= -180 - 27x + 288 - 27x$

$= 108 - 54x = 0$

CN : $x = \frac{108}{54} = 2$

will this give MAX revenue?

① $R'' = -54$

$R''(2) = -54$ $\frown$ max

②

$\xrightarrow[\ 2\ ]{+ \quad | \quad -} R'$

max.

wait 2 weeks to pick pears