NAME KEY

Math 12
Test 2
Fall 2011

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Find and identify all extrema and inflection points of the graph of $f(x) = x^3 - 12x + 20$. Do not sketch the graph.

$f'(x) = 3x^2 - 12 = 0$

$3x^2 = 12$

$x = \pm 2$

$\oplus \quad -2 \quad \ominus \quad 2 \quad \oplus \quad \rightarrow f'$

$f''(x) = 6x = 0$

$x = 0$

$\ominus \quad 0 \quad \oplus \quad \rightarrow f''$

maximum at $(-2, 36)$

minimum at $(2, 4)$

inflection point at $(0, 20)$

2. For the following functions, find all horizontal and vertical asymptotes (remember that an asymptote is a LINE, not a number). If there are no asymptotes, say so.

a) $f(x) = 2 + \frac{1}{x^2} = \frac{2x^2+1}{x^2}$

HA: $y = 2$

VA: $x = 0$

b) $f(x) = \frac{\sqrt[3]{8x^6} - 5}{3x^2+1}$ like $\frac{2x^2}{3x^2}$

$\rightarrow 3x^2 + 1 = 0$, no solns

HA: $y = 2/3$

VA: none

c) $f(x) = \frac{x-3}{x^2 - 5x + 6} = \frac{x-3}{(x-2)(x-3)}$

HA: $y = 0$

VA: $x = 3$

(notice $x = 2$ gives a hole)

3. Suppose that at price p, demand for a certain product is given by $q(p)=\sqrt{2500-p^2}$ when price is a positive value less than \$50.

a) Find the price elasticity of demand when price is \$30. Is demand elastic or inelastic at this price?

$$E(p) = \frac{p}{q}\cdot q' = \frac{p}{\sqrt{2500-p^2}}\cdot\frac{1}{2}\left(2500-p^2\right)^{-1/2}(-2p)$$

$$E(30) = \frac{-900}{2500-900} = \frac{-900}{1600} = -\frac{9}{16}$$ demand is <u>inelastic</u>

b) Find the price elasticity of demand when price is \$40. Is demand elastic or inelastic at this price?

$$E(40) = \frac{-1600}{2500-1600} = \frac{-16}{9}$$ demand is <u>elastic</u>.

c) If price moves up from \$30 to \$40, what happens to the elasticity of demand, as shown in (a) and (b)? Explain why elasticity can change depending on price.

As the price increases, demand becomes more elastic. This makes sense, because if an item becomes more and more expensive, consumers become more likely to think of it as a luxury and give up purchasing it. At a high enough price, almost all goods become luxuries.

4. Given the function $f(x)=\dfrac{x}{(2x-1)^3}$, list the intervals where f is increasing, where it is decreasing, where it is concave up and where it is concave down. Do not sketch the graph.

$$f'(x) = \frac{(2x-1)^3 - x(3)(2x-1)^2(2)}{(2x-1)^6} = \frac{2x-1-6x}{(2x-1)^4} = \frac{-4x-1}{(2x-1)^4}$$

$x = -1/4,\ 1/2$

f': $\oplus$ | $-1/4$ | $\ominus$ | $1/2$ | $\ominus$

$$f''(x) = \frac{-4(2x-1)^4 - (-4x-1)(4)(2x-1)^3(2)}{(2x-1)^8} = \frac{-8x+4-8(-4x-1)}{(2x-1)^5} = \frac{24x+12}{(2x-1)^5}$$

$x = -1/2,\ 1/2$

f'': $\oplus$ | $-1/2$ | $\ominus$ | $1/2$ | $\oplus$

increasing on $(-\infty, -1/4)$
decreasing on $(-1/4, 1/2)\cup(1/2, \infty)$
concave up on $(-\infty, -1/2)\cup(1/2, \infty)$
concave down on $(-1/2, 1/2)$

5. The student radio station, KMNR, has surveyed the listening habits of students between 5:00 pm and midnight. The percentage of the student population listening to the station x hours after 5:00 pm is $f(x)=\frac{1}{8}\left(-2x^3+27x^2-108x+240\right)$. At what time is the smallest percentage of the student population listening, and at what time is the largest percentage listening?

x is in [0, 7] (5:00 → 12:00)

$f'(x) = \frac{1}{8}(-6x^2+54x-108)$

$= -\frac{3}{4}(x^2-9x+18)$

$= -\frac{3}{4}(x-3)(x-6)$

$x = 3, 6$

$f(0) = \frac{1}{8}(240) = 30$

$f(3) = \frac{1}{8}(105) = 13.125$

$f(6) = \frac{1}{8}(132) = 16.5$

$f(7) = \frac{1}{8}(121) = 15.125$

smallest percentage is when x=3, at 8:00 pm.
largest percentage is when x=0, at 5:00 pm.

6. Sketch the graph of a function $f(x)$ so that all conditions below are satisfied. Be sure your graph is big enough so I can see it and it is properly labeled.

a) $f(x)$ is defined for all x except $x=2$.

b) $f'(x)<0$ when $x<2$ and when $x>4$, but $f'(x)>0$ when $2<x<4$.

c) $f''(x)<0$ when $0<x<2$ and when $2<x<5$, but $f''(x)>0$ when $x<0$ and when $x>5$.

d) $\lim_{x\to\infty} f(x)=0$.

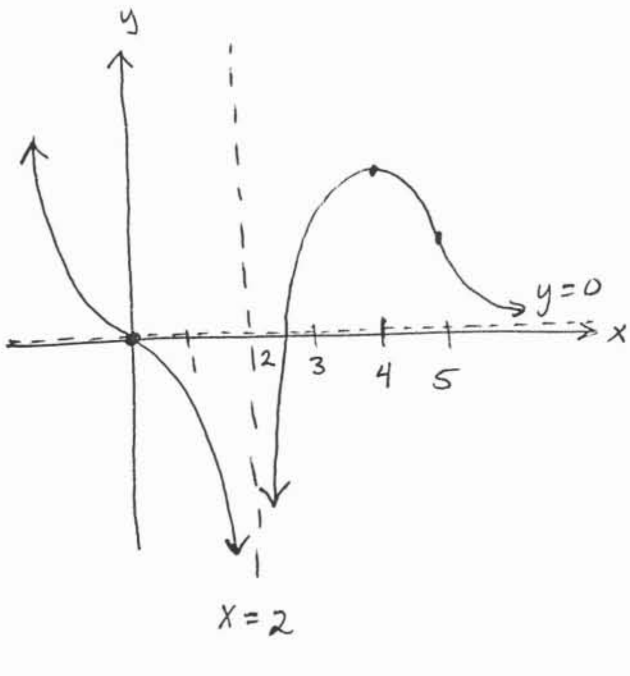
a) asymp or hole when x= 2

b) ⊖ | ⊕ | ⊖ → f′ (2, 4)

c) ⊕ | ⊖ | ⊖ | ⊕ → f″ (0, 2, 5)

Combo: dec up | dec down | dec down | inc down | dec up (0, 2, 4, 5)

d) asymp y=0 on the right side



7. Find the equation of the line tangent to $x\sqrt{y+1} = y\sqrt{x+1}$ at the point (3,3).

$$x(y+1)^{1/2} = y(x+1)^{1/2}$$

$$(1)(y+1)^{1/2} + (x)(\tfrac{1}{2})(y+1)^{-1/2}(y') = (y')(x+1)^{1/2} + (y)(\tfrac{1}{2})(x+1)^{-1/2}$$

at $x=3$, $y=3$, we have

$$4^{1/2} + \tfrac{3}{2}(4)^{-1/2}y' = y'(4)^{1/2} + \tfrac{3}{2}(4)^{-1/2}$$

$$2 + \tfrac{3}{4}y' = 2y' + \tfrac{3}{4}$$

$$\tfrac{5}{4} = \tfrac{5}{4}y'$$

$$y' = 1.$$

<u>Line</u>: $y - 3 = 1(x-3)$ or $x = y$

8. A manufacturing company has total fixed costs of \$1200, material and labor costs combined are \$2 per unit, and the demand equation is $p = \dfrac{100}{\sqrt{q}}$, where p is the price per unit and q is the number of units. How many units should be produced in order to maximize profit?

Profit = Revenue − Cost

= price · quantity − Cost

$$P = (100q^{-1/2})(q) - 1200 - 2q$$

$$P = 100q^{1/2} - 1200 - 2q$$

$$P' = 50q^{-1/2} - 2 = 0$$

$$q^{-1/2} = \frac{1}{25} = \frac{1}{\sqrt{q}}$$

$$\sqrt{q} = 25$$

$$q = 625$$

must be sure this gives a max, not a min:

<u>method ①</u>:

max

$+ \quad | \quad - \longrightarrow P'$

625

<u>method ②</u>:

$$P'' = -25q^{-3/2}$$

$$P''(625) = \frac{-25}{(\sqrt{625})^3} < 0$$

concave down, ⌒ max

$q = 625$ will maximize profit.