NAME KEY

Math 12
Test 2
Spring 2012

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Given the function $f(x) = x^5 - 5x$, list the intervals where f is increasing, where it is decreasing, where it is concave up and where it is concave down. Find all extrema and inflection points. Do not sketch the graph.

$f'(x) = 5x^4 - 5$

$= 5(x^4 - 1)$

$= 5(x^2-1)(x^2+1)$

$= 5(x-1)(x+1)(x^2+1)$

CN : $x = 1, -1$

$\oplus$ -1 $\ominus$ 1 $\oplus$ $\to f'$

$f''(x) = 20x^3$

IN : $x = 0$

$\ominus$ 0 $\oplus$ $\to f''$

increasing on $(-\infty, -1) \cup (1, \infty)$

decreasing on $(-1, 1)$

max $(-1, 4)$

min $(1, -4$

conc up $(0, \infty)$

conc down $(-\infty, 0)$

Infl. pt $(0, 0)$

2. For the following functions, find all horizontal and vertical asymptotes (remember that an asymptote is a LINE, not a number). If there are no asymptotes, say so.

a) $f(x) = \dfrac{3x^2}{x^2-16} = \dfrac{3x^2}{(x+4)(x-4)}$

VA : $x = 4$, $x = -4$

HA : $y = 3$

b) $f(x) = \dfrac{x^2-2x-3}{x^3-5x^2+6x} = \dfrac{(x-3)(x+1)}{x(x-3)(x-2)}$

VA : $x = 0$, $x = 2$

(hole at $x = 3$)

HA : $y = 0$

c) $f(x) = \dfrac{4}{x^2+1} + 7 = \dfrac{4 + 7(x^2+1)}{x^2+1}$

$= \dfrac{7x^2+11}{x^2+1}$

VA : none

HA : $y = 7$

3. Suppose that at price p, demand for a certain product is given by $q(p) = 10{,}000 - 500p$ when price is a positive value less than \$20.

a) Find the price elasticity of demand when price is \$5. Is demand elastic or inelastic at this price?

$E(p) = \frac{p}{q} \cdot q' = \frac{p}{10000 - 500p} \cdot (-500)$

$E(5) = \frac{-2500}{10000 - 2500} = \frac{-1}{3}$

$|E(5)| = \frac{1}{3} < 1$, demand is inelastic

b) Suppose the price of \$5 is increased to \$5.15. What is the percentage increase in price?

$\%\text{ increase} = \frac{\text{amt of increase}}{\text{orig amt}} \cdot 100\% = \frac{0.15}{5.00} \cdot 100\% = 3\%$

c) If price is increased from \$5 to \$5.15, use (a) and (b) to determine the new demand amount.

by (a), if price ↑ 1%, demand ↓ $\frac{1}{3}$%.
using (b), if price ↑ 3%, demand ↓ 1%.
old demand $= q(5) = 7500$. 1% of this is 75, so
new demand $= 7500 - 75 = 7425$

4. For the function $f(x) = \dfrac{4x}{x^2+1}$, list the intervals where f is concave up and where it is concave down. Find all inflection points. Find all asymptotes. If any of these items do not exist, say so.

$$f'(x) = \frac{4(x^2+1) - (4x)(2x)}{(x^2+1)^2} = \frac{4x^2+4-8x^2}{(x^2+1)^2} = \frac{4-4x^2}{(x^2+1)^2}$$

$$f''(x) = \frac{(-8x)(x^2+1)^2 - (4-4x^2)(2)(x^2+1)(2x)}{(x^2+1)^4}$$

$$= \frac{-8x(x^2+1) - (4-4x^2)(4x)}{(x^2+1)^3} = \frac{-8x^3-8x-16x+16x^3}{(x^2+1)^3}$$

$$= \frac{8x^3-24x}{(x^2+1)^3} = \frac{8x(x^2-3)}{(x^2+1)^3}$$

IN: $x = 0, \pm\sqrt{3}$

f'': ⊖ on $(-\infty,-\sqrt{3})$, ⊕ on $(-\sqrt{3},0)$, ⊖ on $(0,\sqrt{3})$, ⊕ on $(\sqrt{3},\infty)$

$-\sqrt{3}$ $\quad 0 \quad$ $\sqrt{3}$

conc up on $(-\sqrt{3},0) \cup (\sqrt{3},\infty)$
conc down on $(-\infty,-\sqrt{3}) \cup (0,\sqrt{3})$
inf. pts $(-\sqrt{3},-\sqrt{3}), (0,0), (\sqrt{3},\sqrt{3})$
VA: none
HA: $y=0$

5. Find the absolute extrema of $f(x) = x^4 - 8x^2 + 16$ on the interval $[1,3]$.

$f'(x) = 4x^3 - 16x = 4x(x^2-4) = 4x(x+2)(x-2)$

$\underline{CN}$: $x = 0, -2, 2$

not in interval

$f(0) = \cancel{16}$

$f(2) = 16 - 32 + 16 = 0 \longrightarrow$ abs min $(2,0)$

$f(1) = 1 - 8 + 16 = 9$

$f(3) = 81 - 72 + 16 = 25 \longrightarrow$ abs max $(3,25)$

6. Sketch the graph of a function $f(x)$ so that all conditions below are satisfied. Be sure your graph is big enough so I can see it and it is properly labeled.

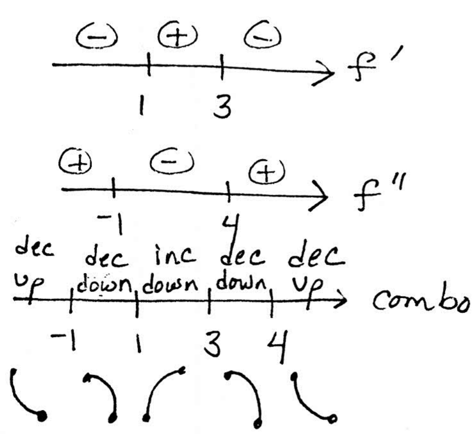
a) $f(x)$ is defined for all x except $x = 1$. $\longrightarrow$ asymp or hole at $x=1$

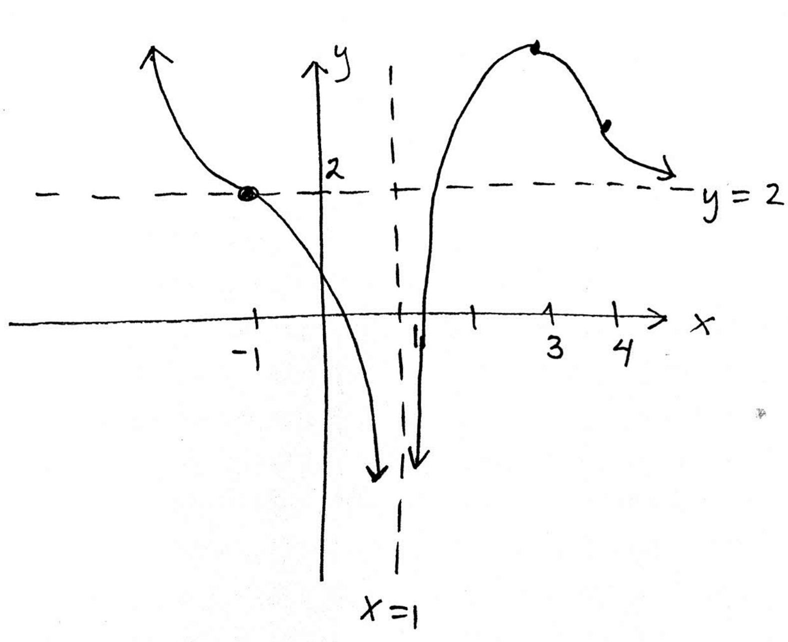
b) $f'(x) > 0$ when $1 < x < 3$. inc

c) $f''(x) > 0$ when $x > 4$ and when $x < -1$. up

d) $f(x)$ has an inflection point at (-1,2).

e) $\lim_{x \to \infty} f(x) = 2$. $\longrightarrow$ a symp $y=2$





7. Find the equation of the line tangent to $x^4 - 2x^3y^2 = 1 - y^3$ at the point (2,1).

$$4x^3 - (6x^2y^2 + 4x^3yy') = -3y^2y'$$

Fill in $x=2, y=1$

$$32 - (24 + 32y') = -3y'$$

$$32 - 24 = 32y' - 3y'$$

$$8 = 29y'$$

$$y' = m = \frac{8}{29}$$

Line: $y - 1 = \frac{8}{29}(x-2)$

8. A company that makes iPad keyboard cases finds that if q packages of cases are produced in an hour, the *average* cost per package is $\bar{c} = 2q^2 - 36q + 210 - \frac{200}{q}$, where the number of packages that can be produced per hour is restricted to $2 \le q \le 10$. How many packages should be produced (within the restrictions) in order to maximize total cost? What is the minimum total cost in an hour?

Total cost $= C = \bar{c} \cdot q = 2q^3 - 36q^2 + 210q - 200$

$$C' = 6q^2 - 72q + 210$$

$$= 6(q^2 - 12q + 35) = 6(q-5)(q-7)$$

CN: $q = 5, 7$

$C(2) = 16 - 144 + 420 - 200 = 92 \longrightarrow$ produce 2 pkgs per hour at a minimum total cost of \$92. per hour

$C(5) = 250 - 900 + 1050 - 200 = 200$

$C(7) = 686 - 1764 + 1470 - 200 = 192$

$C(10) = 2000 - 3600 + 2100 - 200 = 300$