NAME KEY

Math 12
Test 2
Summer 2013

You have 60 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Suppose $f(x)=\dfrac{x^2}{x-2}$. Find all critical numbers, list the intervals of increase and decrease, and tell whether each critical number will result in a maximum, a minimum, or neither. You do not need to find the y-values for the extrema.

$$f'(x) = \frac{2x(x-2)-x^2}{(x-2)^2} = \frac{2x^2-4x-x^2}{(x-2)^2} = \frac{x^2-4x}{(x-2)^2} = \frac{x(x-4)}{(x-2)^2}$$

critical numbers: $x = 0, 2, 4$

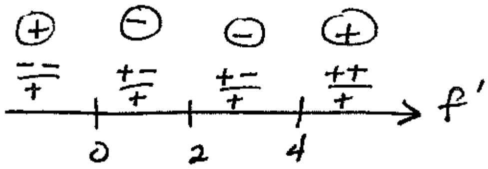
increasing on $(-\infty, 0) \cup (4, \infty)$

decreasing on $(0,2) \cup (2,4)$

$x=0$ gives a maximum

$x=2$ gives neither

$x=4$ gives a minimum



2. For the following functions, find all horizontal and vertical asymptotes (remember that an asymptote is a LINE, not a number). If there are no asymptotes, say so.

(a) $f(x)=\dfrac{2x^2+3x+1}{3x^2-5x+2} = \dfrac{(2x+1)(x+1)}{(3x-2)(x-1)}$

VA: $x = 2/3$, $x=1$

HA: $y = 2/3$

(b) $f(x)=\dfrac{x+2}{x^2-4} = \dfrac{x+2}{(x+2)(x-2)}$

VA: $x=2$
(notice $x=-2$ gives a hole, not an asymp.

HA: $y=0$

(c) $f(x)=x-\dfrac{1}{x} = \dfrac{x^2}{x} - \dfrac{1}{x} = \dfrac{x^2-1}{x}$

VA: $x=0$

HA: none

3. a) Suppose that $q(p) = 200 - 2p^2$ units of a product are demanded when the price is set at p dollars per unit, assuming $0 \le p \le 250$. Calculate the elasticity of demand when $p = 6$. Interpret (write a sentence about) your answer in terms of percentage changes, and also tell whether the demand for the product is elastic or inelastic at this point.

$$E(p) = \frac{p}{q} \cdot q' = \frac{p}{200-2p^2} \cdot (-4p)$$

$$E(6) = \frac{6}{200-72}(-24) = \frac{-144}{128} = \frac{-9}{8} = -1.125$$

when price is \$6 and increases 1%, demand will go down 1.125%.

Since $|E(6)| = 1.125 > 1$, demand for this product is elastic.

b) Give an example of a product whose demand function would, in general, behave as in (a). Explain why your example behaves this way.

Various examples. Should be a somewhat luxury item that costs about \$6. Movie, lunch out, going for ice cream ...

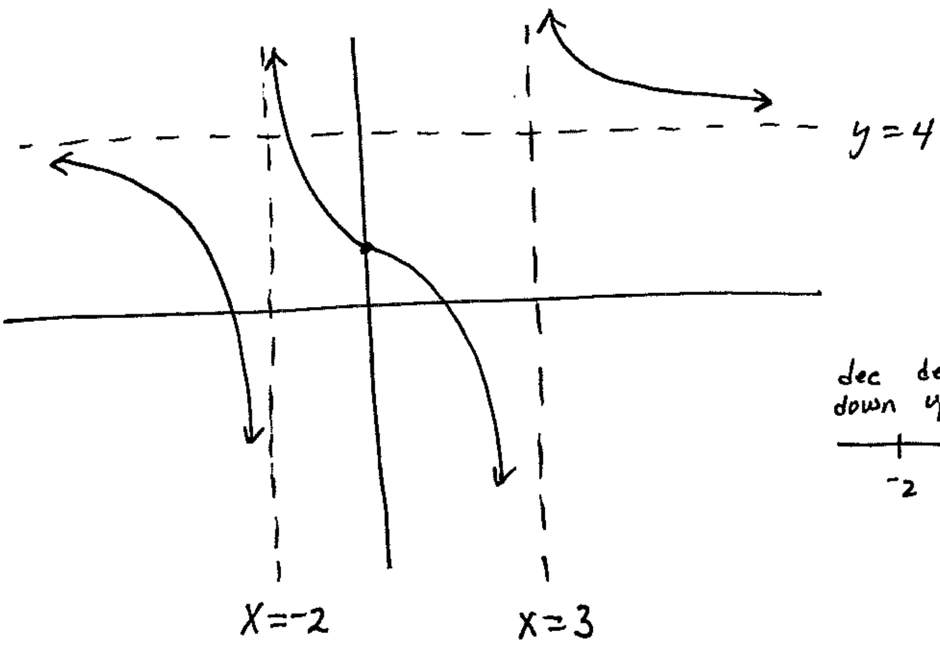
4. Sketch a nice BIG graph of a function with all the properties listed below. Make sure your graph is clearly labeled.

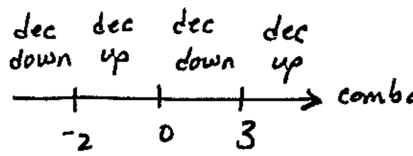
a) $f'(x) < 0$ for all values of x except $x = -2$ and $x = 3$

b) $f''(x) < 0$ on the intervals $(-\infty,-2) \cup (0,3)$, but $f''(x) \ge 0$ otherwise

c) $f(x)$ is undefined when $x = -2$ and when $x = 3$ $\longrightarrow$ asymptotes or holes

d) $\lim_{x \to -\infty} f(x) = 4$. $\longrightarrow$ horiz asymp at $y = 4$.





5. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x)=x^2(3-2x)^3$

$$f'(x)=(2x)(3-2x)^3+(x^2)(3)(3-2x)^2(-2)$$

(b) $f(x)=\sqrt{\dfrac{1-2x}{3x-2}} = \left(\dfrac{1-2x}{3x-2}\right)^{1/2} = \dfrac{(1-2x)^{1/2}}{(3x-2)^{1/2}}$

$$f'(x)=\frac{1}{2}\left(\frac{1-2x}{3x-2}\right)^{-1/2}\left(\frac{(-2)(3x-2)-(1-2x)(3)}{(3x-2)^2}\right)$$

OR $$f'(x)=\frac{\frac{1}{2}(1-2x)^{-1/2}(-2)(3x-2)^{1/2}-(1-2x)^{1/2}\left(\frac{1}{2}\right)(3x-2)^{-1/2}(3)}{3x-2}$$

6. Find y' if $x^2y-2xy^3+6=2x+2y$.

$$2xy+x^2y'-(2y^3+2x\cdot 3y^2y')=2+2y'$$

$$x^2y'-6xy^2y'-2y'=2-2xy+2y^3$$

$$y'(x^2-6xy^2-2)=2-2xy+2y^3$$

$$y'=\frac{2-2xy+2y^3}{x^2-6xy^2-2}$$

7. Find the absolute minimum and absolute maximum *points* of $f(x) = \frac{1}{3}x^3 - 9x + 2$ on the interval $0 \le x \le 2$.

$f'(x) = x^2 - 9 = (x+3)(x-3)$

CN : $x = 3, -3$ ← not in interval, so don't bother checking

endpts : $0, 2$.

$f(0) = 0 - 0 + 2 = 2$ ← biggest

$f(2) = \frac{8}{3} - 18 + 2$

$= \frac{8}{3} - \frac{48}{3} = -\frac{40}{3}$ ← smallest

absolute max $(0, 2)$

absolute min $(2, -\frac{40}{3})$

8. Suppose that your student organization has been collecting glass bottles to be recycled as a fundraiser. In the past 80 days, 24,000 pounds of glass has been collected. Currently, the recycling company will pay \$1 for each 100 pounds of glass. However, the recycling company has incurred extra costs, and plans to begin reducing by 1 cent each day the price it will pay for 100 pounds of glass. Your club can continue to collect more glass at the same rate they have been each day, but they can only make one trip to the recycling center. When is the best time to conclude this project and deliver the bottles to the recycling center? (Be sure you're getting a MAXIMUM payment. Also, you might be happier if you work in pennies because you won't have decimals).

Start with 240 hundred pounds at \$1 = 100 cents per hundred pounds

Let x = # extra days

Each day we collect $\frac{240}{80}$ hundred pounds = 3 hundred pounds

Each day price per hundred pounds goes down 1 cent.

P = payment = (# hundred pounds)(price per hundred pounds)

$P = (240 + 3x)(100 - x)$ cents

$P = 24000 + 300x - 240x - 3x^2$

$P = -3x^2 + 60x + 24000$

$P' = -6x + 60 = 0$ CN : $x = 10$ days

$P'' = -6$

$P''(10) = -6 < 0$ max } OR { $\underset{10}{+ \mid -}$ P'

max when $x = 10$ days