NAME Key

Math 12
Test 3
Fall 2010

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 6 of the following 7 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves tomorrow.

1. Solve $y' = \sqrt{y}e^x - \sqrt{y}$ if $y = 1$ when $x = 0$.

$$\frac{dy}{dx} = \sqrt{y}\,(e^x - 1)$$

$$y^{-1/2}\,dy = (e^x - 1)\,dx$$

$$2y^{1/2} = e^x - x + C$$

$$2(1)^{1/2} = e^0 - 0 + C$$

$$2 = 1 + C$$

$$1 = C$$

$$\sqrt{y} = \frac{1}{2}e^x - \frac{1}{2}x + \frac{1}{2}$$

$$y = \left(\frac{1}{2}e^x - \frac{1}{2}x + \frac{1}{2}\right)^2$$

2. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = \dfrac{e^{4x}}{x^2 - 2}$

$$f'(x) = \frac{(4e^{4x})(x^2-2) - (e^{4x})(2x)}{(x^2-2)^2}$$

(b) $f(x) = ln(e^x - 3x)$

$$f'(x) = \left(\frac{1}{e^x - 3x}\right)\left(e^x - 3\right)$$

3. Suppose you want to have \$15,000 saved for a wedding 3 years from now. You and your fiancé already have a commitment from your future in-laws to contribute \$8000 at the time of the wedding. How much should you invest now in one lump sum to make up the difference if you can earn an annual interest rate of 12% compounded monthly?

$$\begin{array}{r} 15000 \\ -8000 \\ \hline 7000 \end{array} = \text{amt needed at end}$$

$B = 7000$

$r = .12$

$K = 12$

$t = 3$ years

$$B = P\left(1 + \frac{r}{K}\right)^{Kt}$$

$$7000 = P\left(1 + \frac{.12}{12}\right)^{12(3)} = P(1.01)^{36}$$

$$P = \frac{7000}{(1.01)^{36}} \approx \$4892.47$$

4. a) Find x if $ln(x^3) - ln(x) = 2$

$$\ln\left(\frac{x^3}{x}\right) = 2$$

$$\ln(x^2) = 2 \xrightarrow{OR} 2\ln x = 2$$

$$\ln x = 1$$

$$x^2 = e^2$$

$$x = e^1 = e$$

$$x = e$$

b) If $log_2 x = 2$, $log_2 y = -3$, and $log_2 z = 6$, find $log_2\left(\frac{x^3}{y\sqrt{z}}\right)$.

$$\log_2\left(\frac{x^3}{y z^{1/2}}\right) = \log_2 x^3 - \left(\log_2 y z^{1/2}\right)$$

$$= \log_2 x^3 - \log_2 y - \log_2 z^{1/2}$$

$$= 3\log_2 x - \log_2 y - \tfrac{1}{2}\log_2 z$$

$$= 3(2) - (-3) - \tfrac{1}{2}(6) = 6 + 3 - 3 = 6$$

5. For the function $f(x) = ln(1 + x^2)$, list all intervals of increase and decrease, all maximum and minimum *points*, intervals where the function is concave up and concave down, all inflection *points*, and all asymptotes (or say there are none). Then sketch the graph of the function.

$$f'(x) = \frac{1}{1+x^2}(2x) = \frac{2x}{1+x^2}$$

<u>crit #</u>: $x = 0$ $\qquad \xrightarrow{\;-\;\;|_{0}\;\;+\;} f'$

$$f(0) = \ln(1) = 0$$

$$f''(x) = \frac{(2)(1+x^2) - (2x)(2x)}{(1+x^2)^2}$$

$$= \frac{2 + 2x^2 - 4x^2}{(1+x^2)^2} = \frac{2-2x^2}{(1+x^2)^2}$$

$$= \frac{-2(x^2-1)}{(1+x^2)^2} = \frac{-2(x+1)(x-1)}{(1+x^2)^2}$$

<u>Inf #'s</u>: $x = 1, -1$ $\qquad \xrightarrow{\;-\;\;|_{-1}\;\;+\;\;|_{1}\;\;-\;} f''$

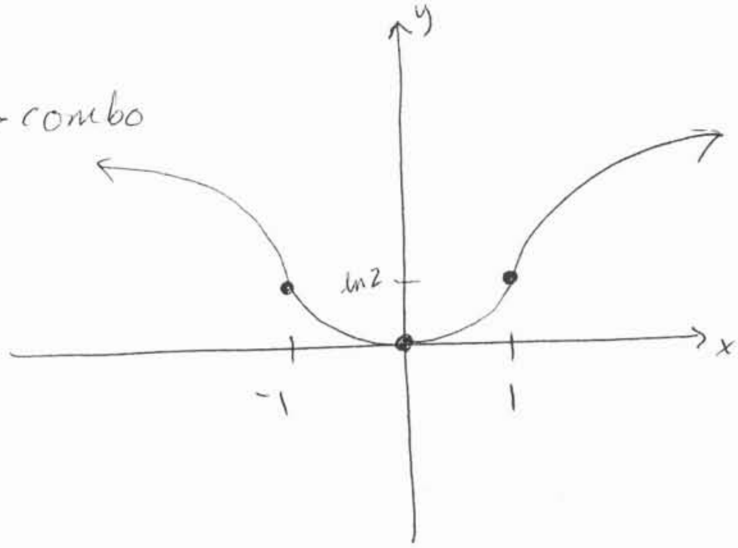
inc on $(0, \infty)$
dec on $(-\infty, 0)$
min $(0,0)$
no max
conc up on $(-1,1)$
conc down on $(-\infty,-1) \cup (-1,\infty)$
inf pts
$(1, \ln 2)$
$(-1, \ln 2)$
≈ 0.69

No VA
No HA

f is defined for all x, no VA
If $x \to \pm\infty$, $f(x) \to \infty$, no HA

dec down	dec up	inc up	inc down

$\xrightarrow{\;|_{-1}\;\;|_{0}\;\;|_{1}\;}$ combo



6. Evaluate the following integrals:

a) $\int\left(2\sqrt{x}-3\sqrt[4]{x}\right)dx$ $= \int(2x^{1/2}-3x^{1/4})\,dx$

$= 2\left(\frac{2}{3}x^{3/2}\right)-3\left(\frac{4}{5}x^{5/4}\right)+C$

$= \frac{4}{3}x^{3/2}-\frac{12}{5}x^{5/4}+C$

b) $\int\frac{2x^3+3x}{x^4+3x^2+7}\,dx$ $= \int\frac{1/2\,du}{u} = \frac{1}{2}\ln|u|+C$

$= \frac{1}{2}\ln|x^4+3x^2+7|+C$

$u = x^4+3x^2+7$

$du = (4x^3+6x)\,dx$

$= 2(2x^3+3x)dx$

$\frac{1}{2}du = (2x^3+3x)dx$

7. Solve $\int e^{-x}(x+1)\,dx$

$u = x+1$ $\quad dv = e^{-x}dx$

$du = dx$ $\quad v = \int e^{-x}dx = -e^{-x}$

$uv - \int v\,du$

$\int e^{-x}(x+1)\,dx = -e^{-x}(x+1)+\int e^{-x}dx$

$= -e^{-x}(x+1)-e^{-x}+C$