NAME $\underline{\quad KEY \quad}$

Math 12
Test 3
Fall 2011

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 6 of the following 7 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves tomorrow.

1. Solve $x^2y' + \frac{1}{y^2} = 0$ if $y = 2$ when $x = 1$.

$$x^2 \frac{dy}{dx} = \frac{-1}{y^2}$$

$$x^2\, dy = \frac{-1}{y^2}\, dx$$

$$y^2\, dy = \frac{-1}{x^2}\, dx$$

$$\int y^2\, dy = -\int x^{-2}\, dx$$

$$\frac{1}{3}y^3 = x^{-1} + C$$

To find C, fill in $x=1$, $y=2$:

$$\frac{1}{3}(2)^3 = 1 + C$$

$$\frac{8}{3} = 1 + C$$

$$C = \frac{5}{3}$$

$$\frac{1}{3}y^3 = x^{-1} + \frac{5}{3}$$

$$y^3 = 3x^{-1} + 5$$

$$y = \sqrt[3]{3x^{-1}+5}$$

2. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = \frac{4e^{3x}}{xe^{x-1}}$

$$f'(x) = \frac{(12e^{3x})(xe^{x-1}) - (4e^{3x})\left((1)(e^{x-1}) + (x)(e^{x-1})(1)\right)}{(x\,e^{x-1})^2}$$

(b) $f(x) = \frac{\ln x}{\sqrt{x}} = \frac{\ln x}{x^{1/2}}$

$$f'(x) = \frac{(\frac{1}{x})(x^{1/2}) - (\ln x)(\frac{1}{2}x^{-1/2})}{x}$$

3. Find the balance in an investment account of \$4000 for 5 years at the annual rate of 11% compounded monthly.

$$B = P\left(1+\frac{r}{k}\right)^{kt}$$

$$B = 4000\left(1+\frac{.11}{4}\right)^{4(5)}$$

$$= 4000(1.0275)^{20}$$

$$\approx \$6881.71$$

4. a) Simplify $\log_2\left[\ln\left(\sqrt{7+e^2}+\sqrt{7}\right)+\ln\left(\sqrt{7+e^2}-\sqrt{7}\right)\right]$.

$$= \log_2\left(\ln\left(\sqrt{7+e^2}+\sqrt{7}\right)\cdot\left(\sqrt{7+e^2}-\sqrt{7}\right)\right)$$

$$= \log_2\left(\ln\left(\cancel{7}+e^2-\cancel{7}\right)\right) = \log_2(2) = 1$$

b) Solve for x: $\log_x(2x+3) = 2$.

$$2x+3 = x^2$$

$$x^2-2x-3=0$$

$$(x-3)(x+1)=0$$

$x = 3, \cancel{-1} \leftarrow$ can't be base!

$x = 3$

c) Solve for x: $3^{4x} = 9^{x+1}$.

$$3^{4x} = (3^2)^{x+1}$$

$$3^{4x} = 3^{2x+2}$$

$$4x = 2x+2$$

$$2x = 2$$

$$x = 1$$

5. For the function $f(x) = \frac{e^x + e^{-x}}{2}$, list all intervals of increase and decrease, all maximum and minimum *points*, intervals where the function is concave up and concave down, all inflection *points*, and all asymptotes (or say there are none). Then sketch the graph of the function.

$$f'(x) = \frac{e^x - e^{-x}}{2} = 0$$

$$e^x = e^{-x}$$

$$x = -x$$

CN: $x = 0$

f': $-$ on the left of 0, $+$ on the right of 0

$$f(0) = \frac{e^0 + e^0}{2} = \frac{1+1}{2} = 1$$

min $(0,1)$

$$f''(x) = \frac{e^x + e^{-x}}{2} = 0$$

$$e^x + e^{-x} = 0$$

$$\underbrace{e^x}_{\text{always}+} = \underbrace{-e^{-x}}_{\text{always}-}$$

no Inf. #'s.

f'': $+$

Always defined, so NO VA.

If $x \to \infty$, $y \to \frac{e^{big} + \cancel{e^{-big}}^{\to 0}}{2} \to \infty$

If $x \to -\infty$, $y \to \frac{\cancel{e^{-big}}^{\to 0} + e^{big}}{2} \to \infty$

NO HA.

inc on $(0, \infty)$

dec on $(-\infty, 0)$

min at $(0,1)$

no max

concave up on $(-\infty, \infty)$

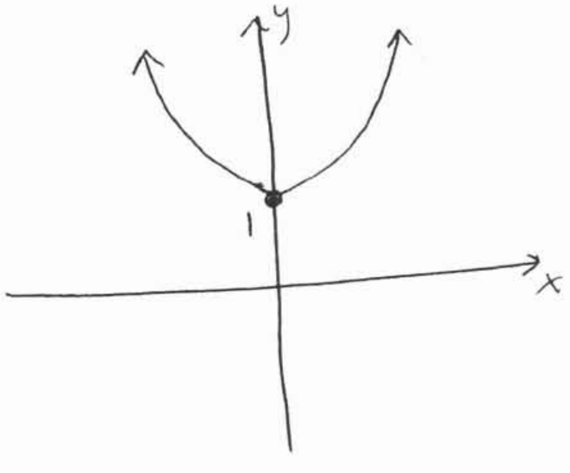
never concave down

no inflection points

VA: none

HA: none

combo: dec, up on the left of 0; inc, up on the right of 0



6. Evaluate the following integrals:

a) $\int (x^e + e^x)dx$ $= \frac{1}{e-1} x^{e-1} + e^x + C$

b) $\int e^{x^2+\ln x}\, dx$ $= \int e^{x^2} \cdot e^{\ln x}\, dx$ $= \int e^{x^2} \cdot x\, dx$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2} du = x\,dx$

$= \int e^u \left(\frac{1}{2} du\right)$

$= \frac{1}{2} e^u + C$

$= \frac{1}{2} e^{x^2} + C$

7. Solve $\int \frac{\ln x}{x^2}\, dx$

Let $u = \ln x$ $\qquad dv = x^{-2} dx$

$du = \frac{1}{x} dx$ $\qquad v = \int x^{-2} dx = -x^{-1}$

$$\int \frac{\ln x}{x^2}\, dx = uv - \int v\,du$$

$$= -x^{-1} \ln x - \int (-x^{-1})\left(\frac{1}{x}\right) dx$$

$$= -\frac{1}{x} \ln x + \int x^{-2} dx$$

$$= \frac{-1}{x} \ln x - \frac{1}{x} + C$$