NAME Key

Math 12
Test 3
Spring 2011

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 6 of the following 7 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves tomorrow.

1. Solve $y' = \frac{y^2}{x}$ if $y = 5$ when $x = e$.

$$\frac{dy}{dx} = \frac{y^2}{x}$$

$$\frac{dy}{y^2} = \frac{1}{x}\,dx$$

$$\int y^{-2}\,dy = \int \frac{1}{x}\,dx$$

$$-y^{-1} = \ln|x| + C$$

$$-\frac{1}{y} = \ln|x| + C$$

$$-\frac{1}{5} = \ln e + C$$

$$-\frac{1}{5} = 1 + C$$

$$\frac{4}{5} = C$$

$$-\frac{1}{y} = \ln|x| + \frac{4}{5}$$

$$y = \frac{1}{-\ln|x| - 4/5}$$

2. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = x^2 \ln x - \frac{1}{2}x^2 + e^{x^2} + 5$

$$f'(x) = 2x \cdot \ln x + x^2\left(\frac{1}{x}\right) - x + e^{x^2}(2x)$$

(b) $f(x) = e^{-2x} - x\ln x + x - 7$

$$f'(x) = -2e^{-2x} - \left(\ln x + x\left(\frac{1}{x}\right)\right) + 1$$

3. An art collection is currently worth \$20,000, and its value is increasing at a rate of $300\sqrt{t}$ dollars per year after t years. Find a formula for the value of the collection at any time. Then find the value after 25 years. Would the owner be better off financially to keep the collection for the 25 years, or to sell it for the current value and put the money in his savings account instead? Suppose his savings earns annual interest at 1.5% compounded monthly.

$t=0 \quad V=20{,}000$

$\frac{dV}{dt} = 300t^{1/2}$

$dv = 300t^{1/2}dt$

$V = \int 300t^{1/2}dt$

$= 200t^{3/2}+C$

$20{,}000 = 200(0)+C$

$V = 200t^{3/2}+20000$

In 25 years, $V = 200(\sqrt{25})^3+2000$

$= \$45000$

Investing 20000 for 25 yrs at 1.5% monthly gives

$B = P(1+\frac{r}{k})^{kt}$

$= 20000(1+\frac{.015}{12})^{12(25)}$

$= 20000(1.00125)^{300}$

$= \$29{,}093.01$

Better to keep the collection.

4. a) Find x if $\log_x \frac{8}{27} = 3$. $\quad \frac{8}{27} = x^3, \quad x = 2/3$

b) Find x if $\log_2 x - \log_2(x-1) = 1$. $\quad \frac{x}{x-1} = 2, \quad x = 2x-2$

$2 = x$

c) Find x if $\log_4 x = \frac{5}{2}$. $\quad x = 4^{5/2} = 2^5 = 32$

d) Find x if $3^{2x} = 4^{x+1}$.

$2x\ln 3 = (x+1)\ln 4$

$= x\ln 4 + \ln 4$

$2x\ln 3 - x\ln 4 = \ln 4$

$x(2\ln 3 - \ln 4) = \ln 4$

$x = \frac{\ln 4}{2\ln 3 - \ln 4}$

5. For the function $f(x)=\frac{x}{e^x}$, list all intervals of increase and decrease, all maximum and minimum *points*, intervals where the function is concave up and concave down, all inflection *points*, and all asymptotes (or say there are none). Then sketch the graph of the function.

$$f'(x)=\frac{(1)(e^x)-(x)(e^x)}{e^{2x}}=\frac{e^x(1-x)}{e^{2x}}=\frac{1-x}{e^x}$$

CN: $x=1$

f': $+$ on the left of 1, $-$ on the right of 1

$$f''(x)=\frac{(-1)(e^x)-(1-x)(e^x)}{e^{2x}}=\frac{x-2}{e^x}$$

Inf #: $x=2$

f'': $-$ on the left of 2, $+$ on the right of 2

Defined for all x, no VA

As $x\to\infty$, denom gets really big, $y\to 0$, HA $y=0$ (on right side)

as $x\to-\infty$, $y\to-\infty$

combo: inc down | 1 | dec down | 2 | dec up

increasing on $(-\infty,1)$

decreasing on $(1,\infty)$

max at $(1,\frac{1}{e})$

no min.

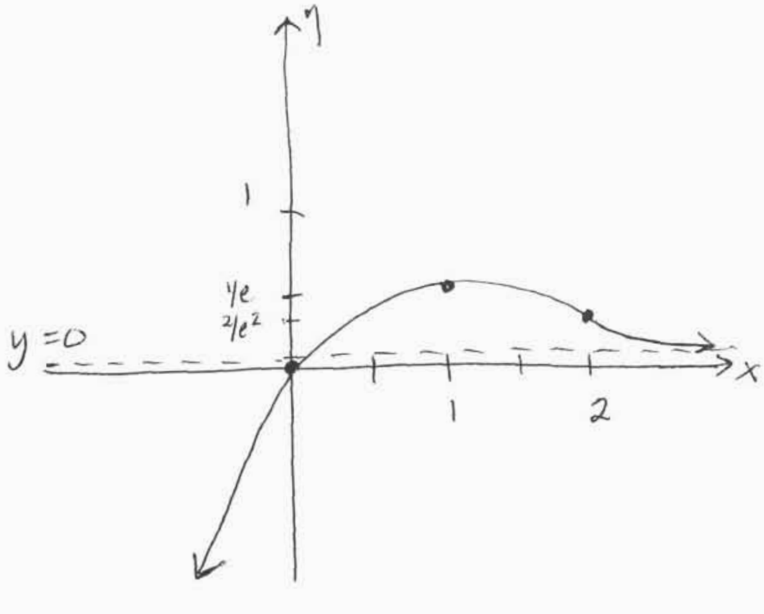
conc up on $(2,\infty)$

conc down on $(-\infty,2)$

inf pt $(2,\frac{2}{e^2})$

VA: none

HA: $y=0$



6. Evaluate the following integrals:

a) $\int \frac{1}{x \ln x} dx \quad = \quad \int \frac{1}{\ln x} \cdot \frac{1}{x} dx \quad = \int \frac{1}{u} du = \ln|u| + C$

$= \ln|\ln x| + C$

Let $u = \ln x$

then $du = \frac{1}{x} dx$

b) $\int \frac{2x^3 + 3x}{x^4 + 3x^2 + 7} dx \longrightarrow = \frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln|u| + C$

$= \frac{1}{2} \ln|x^4 + 3x^2 + 7| + C$

$= \frac{1}{2} \ln(\underbrace{x^4 + 3x^2 + 7}_{\text{(always positive)}}) + C$

Let $u = x^4 + 3x^2 + 7$

then $du = (4x^3 + 6x)dx$

$= 2(2x^3 + 3x)dx$

$\frac{1}{2} du = (2x^3 + 3x)dx$

7. Solve $\int x^2 \ln 2x \, dx$

Let $u = \ln 2x$

$du = \frac{1}{2x} \cdot 2 \, dx$

$= \frac{1}{x} dx$

Let $dv = x^2 dx$

$v = \int x^2 dx$

$v = \frac{1}{3} x^3$

$\int u \, dv = uv - \int v \, du$

$\int x^2 \ln 2x \, dx = (\ln 2x)(\frac{1}{3} x^3) - \int (\frac{1}{3} x^3)(\frac{1}{x} dx)$

$= \frac{1}{3} x^3 \ln 2x - \frac{1}{3} \int x^2 dx$

$= \frac{1}{3} x^3 \ln 2x - \frac{1}{9} x^3 + C$