NAME KEY

Math 12
Test 3
Spring 2012

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 6 of the following 7 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves tomorrow.

1. Solve $y' = \dfrac{xy}{\sqrt{1-x^2}}$ if $y = 2$ when $x = 0$.

$$\frac{dy}{dx} = \frac{xy}{(1-x^2)^{1/2}}$$

$$\frac{dy}{y} = \frac{x}{(1-x^2)^{1/2}}\,dx$$

$$\int \frac{1}{y}\,dy = \int \frac{x}{(1-x^2)^{1/2}}\,dx$$

Let $u = 1-x^2$
then $du = -2x\,dx$

$$-\tfrac{1}{2}\,du = x\,dx$$

$$\ln|y| = \frac{-1}{2}\int u^{-1/2}\,du$$

$$\ln|y| = \frac{-1}{2}\left(2u^{1/2}\right) + C$$

$$= -(1-x^2)^{1/2} + C$$

If $x = 0$, $y = 2$, so

$\ln 2 = -1 + C$, $C = 1 + \ln 2$

$$\ln|y| = -\sqrt{1-x^2} + 1 + \ln 2$$

$$|y| = e^{-\sqrt{1-x^2} + 1 + \ln 2}$$

$$y = \pm 2e^{-\sqrt{1-x^2} + 1}$$

2. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = \dfrac{e^x + x}{\ln x}$

$$f'(x) = \frac{(e^x+1)(\ln x) - (e^x + x)\left(\frac{1}{x}\right)}{(\ln x)^2}$$

(b) $f(x) = \ln\left(\dfrac{e^{2x}}{x^2+1}\right)$

$$f'(x) = \left(\frac{x^2+1}{e^{2x}}\right)\left(\frac{2e^{2x}(x^2+1) - e^{2x}(2x)}{(x^2+1)^2}\right)$$

3. The undergraduate enrollment at S&T was 5155 in the Fall of 2009. In the Fall of 2011, it was 5501. Assuming enrollment grows exponentially, what is the expected enrollment in Fall 2012?

(1) Fall '09 $t=0$ $B=5155$

(2) Fall '11 $t=2$ $B=5501$

(3) Fall '12 $t=3$ $B=?$

Using $B = Pe^{rt}$:

(1) $5515 = Pe^{r \cdot 0} = P$

so now $B = 5155e^{rt}$

(2) $5501 = 5155e^{r(2)}$

$$\frac{5501}{5155} = e^{2r}$$

$$\ln\left(\frac{5501}{5155}\right) = 2r$$

$$\frac{1}{2}\ln\left(\frac{5501}{5155}\right) = r \approx 0.0325$$

$$B \approx 5155e^{0.0325t}$$

(3) $B \approx 5155e^{0.0325(3)}$

≈ 5683

4. a) Simplify $\log_{25}\frac{1}{125}$.

$$\log_{25}\frac{1}{125} = x$$

$$\frac{1}{125} = 25^x$$

$$5^{-3} = 5^{2x}$$

$$-3 = 2x$$

$$x = -3/2$$

b) Simplify $2e^{3\ln 2}$.

$$2e^{3\ln 2} = 2e^{\ln 2^3} = 2e^{\ln 8} = 2(8) = 16$$

c) Solve for x: $\ln(3-x) - \ln(2x-1) = \ln 2$.

$$\ln\left(\frac{3-x}{2x-1}\right) = \ln 2$$

$$\frac{3-x}{2x-1} = 2$$

$$3 - x = 4x - 2$$

$$5 = 5x$$

$x = 1$ ← ok, works in original.

5. For the function $f(x)=\dfrac{6}{1+e^{-x}}$, list all intervals of increase and decrease, all maximum and minimum *points*, intervals where the function is concave up and concave down, all inflection *points*, and all asymptotes (or say there are none). Then sketch the graph of the function, being sure to label appropriately.

$$f'(x)=\frac{(0)(1+e^{-x})-(6)(-e^{-x})}{(1+e^{-x})^2}=\frac{6e^{-x}}{(1+e^{-x})^2}$$ never zero, no CN.

$\xleftrightarrow{\;+\;}$ f'

$$f''(x)=\frac{(-6e^{-x})(1+e^{-x})^2-(6e^{-x})(2)(1+e^{-x})(-e^{-x})}{(1+e^{-x})^4}=\frac{-6e^{-x}(1+e^{-x})+12e^{-x}e^{-x}}{(1+e^{-x})^3}$$

$$=\frac{6e^{-x}(-1-e^{-x}+2e^{-x})}{(1+e^{-x})^3}=\frac{6e^{-x}(e^{-x}-1)}{(1+e^{-x})^3}$$

never zero

$e^{-x}-1=0$

$e^{-x}=1$

$-x=\ln 1=0$

$x=0$

IN : $x=0$ $\quad$ $+$ | $-$ at 0 $\rightarrow f''$

inc up | inc down at 0 $\rightarrow$ combo

conc up on $(-\infty,0)$

conc down on $(0,\infty)$

inc on $(-\infty,\infty)$

no max or min

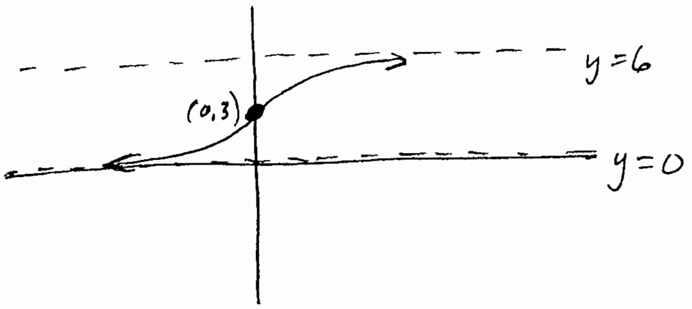
inf pt $(0,3)$ $\qquad f(0)=\frac{6}{1+1}=3$

VA : none, defined for all x

HA : If $x\to\infty$, $\frac{6}{1+e^{-x}}\to\frac{6}{1+\text{small}}\to 6$. $y=6$

If $x\to-\infty$, $\frac{6}{1+e^{-x}}\to\frac{6}{1+\text{big}}\to 0$

$y=0$



6. Evaluate the following integrals:

a) $\int x^3\left(x^2-1\right)^8 dx \longrightarrow = \int x^2 (x^2-1)^8 \cdot x\,dx$

Let $u = x^2 - 1 \rightarrow u+1 = x^2$

then $du = 2x\,dx$

$\frac{1}{2} du = x\,dx$

$$= \frac{1}{2}\int (u+1)u^8\,du$$
$$= \frac{1}{2}\int (u^9 + u^8)\,du$$
$$= \frac{1}{2}\left(\frac{1}{10}(x^2-1)^{10} + \frac{1}{9}(x^2-1)^9\right) + C$$

b) $\int \frac{1}{3x-5}\,dx \longrightarrow = \int \frac{1}{u}\left(\frac{1}{3}du\right) = \frac{1}{3}\ln|3x-5| + C$

Let $u = 3x-5$

then $du = 3\,dx$

$\frac{1}{3}du = dx$

c) $\int\left(\frac{1}{2x} - \frac{2}{x^2} + \frac{3}{\sqrt{x}}\right)dx = \int\left(\frac{1}{2}x^{-1} - 2x^{-2} + 3x^{-1/2}\right)dx$

$$= \frac{1}{2}\ln|x| - 2\left(-x^{-1}\right) + 3\left(2x^{1/2}\right) + C$$
$$= \frac{1}{2}\ln|x| + \frac{2}{x} + 6\sqrt{x} + C$$

7. Solve $\int (x+1)(x+2)^6\,dx$

Let $u = x+1 \qquad dv = (x+2)^6$

$du = dx \qquad v = \int (x+2)^6\,dx = \frac{1}{7}(x+2)^7$

$\int u\,dv = uv - \int v\,du$

$$\int (x+1)(x+2)^6 dx = \frac{1}{7}(x+1)(x+2)^7 - \frac{1}{7}\int (x+2)^7\,dx$$
$$= \frac{1}{7}(x+1)(x+2)^7 - \frac{1}{56}(x+2)^8 + C$$