NAME Key

Math 12
Test 3
Summer 2011

You have 60 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Solve $\frac{dy}{dx} = xy - x$ for y, supposing that $y = 4$ when $x = 0$.

$\frac{dy}{dx} = x(y-1)$

$\frac{dy}{y-1} = x\,dx$

$\int \frac{1}{y-1}\,dy = \int x\,dx$

$u = y-1 \quad \int \frac{1}{u}\,du = \ln|u|$
$du = dy$

$\ln|y-1| = \frac{1}{2}x^2 + C$

Fill in $x=0, y=4$:

$\ln 3 = 0 + C = C$

So now $\ln|y-1| = \frac{1}{2}x^2 + \ln 3$

$|y-1| = e^{\frac{1}{2}x^2 + \ln 3}$

$y - 1 = \pm e^{\frac{1}{2}x^2 + \ln 3}$

$y = 1 \pm e^{\frac{1}{2}x^2 + \ln 3}$

(or $y = 1 \pm 3e^{\frac{1}{2}x^2}$)

2. Suppose your first full-time job out of college has a starting salary of \$53,000 per year. After two years your salary is \$58,000 per year. If your salary grows at an exponential rate, and how much can you expect to be making 15 years after graduation?

① $t = 0 \quad B = 53000$

② $t = 2 \quad B = 58000$

③ $t = 15 \quad B = ?$

$B = Pe^{rt}$

① $53000 = Pe^{r(0)} = P$

$B = 53000\,e^{rt}$

② $58000 = 53000\,e^{r(2)}$

$\frac{58}{53} = e^{2r}$

$\ln\frac{58}{53} = 2r$

$r = \frac{1}{2}\ln\frac{58}{53} \approx 0.04508$

③ $B = 53000\,e^{0.04508(15)}$

$B \approx \$104,211.78$

3. Calculate the following integrals:

a) $\int \frac{8x^3 - 6x^2 - x^4}{3x^3}\,dx$ $= \int \left(\frac{8}{3} - 2x^{-1} - \frac{1}{3}x\right) dx$

$= \frac{8}{3}x - 2\ln|x| - \frac{1}{6}x^2 + C$

b) $\int \frac{(x^2+4)^2}{x^2}\,dx$ $= \int \frac{x^4 + 8x^2 + 16}{x^2}\,dx = \int (x^2 + 8 + 16x^{-2})\,dx$

$= \frac{1}{3}x^3 + 8x - 16x^{-1} + C$

c) $\int \frac{(\ln x)^3}{3x}\,dx$ $= \frac{1}{3}\int u^3\,du = \frac{1}{12}u^4 + C$

$= \frac{1}{12}(\ln x)^4 + C$

$u = \ln x$

$du = \frac{1}{x}\,dx$

4. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = \left(\ln\left(x^3 - 1\right)\right)^2$

$f'(x) = 2(\ln(x^3-1))\left(\frac{1}{x^3-1}\right)(3x^2)$

(b) $f(x) = \frac{\ln x}{e^x + 1}$

$f'(x) = \frac{\left(\frac{1}{x}\right)(e^x+1) - (\ln x)(e^x)}{(e^x+1)^2}$

5. George borrows \$1000 from a loan shark. The loan shark compounds the interest on the loan quarterly. At the end of one year, George owes \$16,000. What annual interest rate was charged by the loan shark?

$P = 1000$
$B = 16000$
$t = 1$
$K = 4$

$B = P\left(1 + \frac{r}{K}\right)^{Kt}$

$16000 = 1000\left(1 + \frac{r}{4}\right)^4$

$16 = \left(1 + \frac{r}{4}\right)^4$

$\sqrt[4]{16} = 1 + \frac{r}{4}$

$2 = 1 + \frac{r}{4}$

$1 = \frac{r}{4}$

$r = 4$

interest rate is 400%

6. Suppose $f(x) = 3 - 5e^{-x}$. Find the intervals where the curve is increasing and where it is decreasing, where it is concave up and where it is concave down, all maxima, minima, inflection points, and asymptotes. If there are none, say so. Then draw a sketch of the graph.

$f'(x) = 5e^{-x}$

CN: none $\xrightarrow{+}$ f'

$f''(x) = -5e^{-x}$

Inf #: none $\xrightarrow{-}$ f''

$f(x)$ is defined for all x, no VA.

If $x \to \infty$, $f(x) \to 3 - 5e^{-\text{big}} \to 3 - 0$

HA: $y = 3$

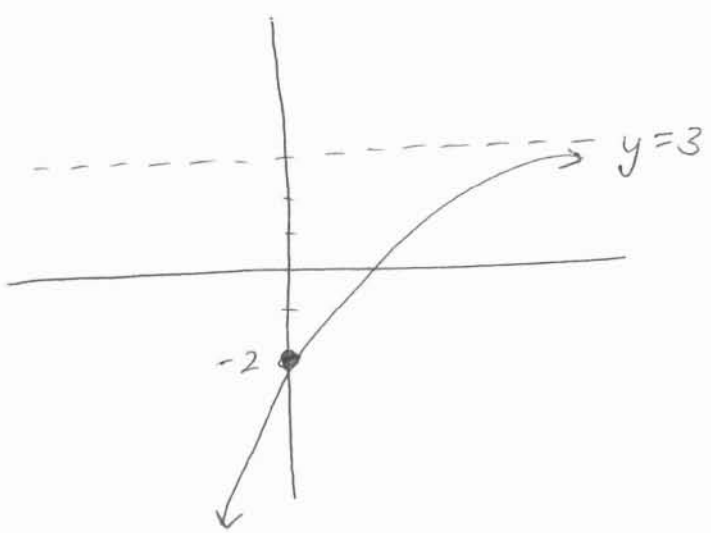
If $x \to -\infty$, $f(x) \to 3 - 5e^{-(-\text{big})}$

$\to 3 - \text{big}$

$\to -\infty$

inc on $(-\infty, \infty)$
dec: (never)
max: none
min: none
conc up: (never)
conc down: $(-\infty, \infty)$
inf pts: none
VA: none
HA: $y = 3$

inc
down
$\longrightarrow$ combo



7. a) Showing all your steps and without using a calculator, simplify $e^{3\ln 4 - \ln 2}$ as much as possible.

$$e^{3\ln 4 - \ln 2} = e^{\ln 4^3 - \ln 2}$$
$$= e^{\ln\left(\frac{4^3}{2}\right)}$$
$$= \frac{4^3}{2}$$
$$= \frac{64}{2} = 32$$

b) Solve $5 = 1 + 4e^{-6x}$ for x.

$$4 = 4e^{-6x}$$
$$1 = e^{-6x}$$
$$\ln 1 = 0 = -6x$$
$$0 = x$$

8. Evaluate $\int \sqrt{x} \ln x \, dx$.

$u = \ln x \qquad dv = x^{1/2} dx$

$du = \frac{1}{x} dx \qquad v = \frac{2}{3} x^{3/2}$

$$uv - \int v\,du = \frac{2}{3} x^{3/2} \ln x - \frac{2}{3} \int x^{3/2} \left(\frac{1}{x}\right) dx$$
$$= \frac{2}{3} x^{3/2} \ln x - \frac{2}{3} \int x^{1/2} dx$$
$$= \frac{2}{3} x^{3/2} \ln x - \frac{4}{9} x^{3/2} + C$$