NAME KEY

Math 12
Test 4
Spring 2012

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 6 of the following 7 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Find the area of the region bounded by the curves $y = 4x$ and $y = x^3 + 3x^2$. Be sure to sketch a graph first!

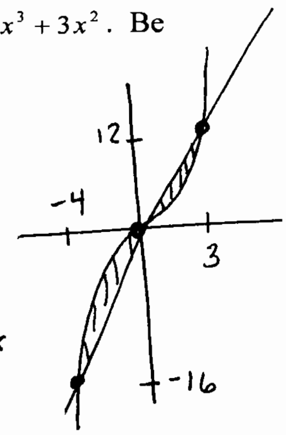
Intersection pts: $x^3 + 3x^2 = 4x$

$x^3 + 3x^2 - 4x = 0$

$x(x^2 + 3x - 4) = 0$

$x(x+4)(x-3) = 0$

$(0,0)$
$(-4,-16)$
$(3,12)$



$$A = \int_{-4}^{0} (x^3 + 3x^2 - 4x)\,dx + \int_0^3 [4x - (x^3 + 3x^2)]\,dx$$

$$= \left[\tfrac{1}{4}x^4 + x^3 - 2x^2\right]_{-4}^{0} + \left[2x^2 - \tfrac{1}{4}x^4 - x^3\right]_0^3$$

$$= \left[0 - \left(\tfrac{256}{4} + -64 - 32\right)\right] + \left[\left(18 - \tfrac{81}{4} - 27\right) - 0\right]$$

$$= -(64 - 64 - 32) + \left(18 - \tfrac{81}{4} - 27\right)$$

$$= 32 + \tfrac{-81}{4} - 9 = 23 - \tfrac{81}{4} = \tfrac{11}{4}$$

2. Find all four second-order partial derivatives of $f(x, y) = x^2 y e^x + 2x^3 y^2$. Do NOT simplify.

$f_x = (2x)(ye^x) + (x^2)(ye^x) + 6x^2y^2$

$f_y = x^2e^x + 4x^3y$

$f_{xx} = (2)(ye^x) + (2x)(ye^x) + (2x)(ye^x) + (x^2)(ye^x) + 12xy^2$

$f_{xy} = 2xe^x + x^2e^x + 12x^2y$

$f_{yy} = 4x^3$

$f_{yx} = 2xe^x + x^2e^x + 12x^2y$

3. Find and classify the critical points of $f(x,y) = x^3 + y^2 - 6xy + 9x + 5y + 2$.

$f_x = 3x^2 - 6y + 9 = 0$

$f_y = 2y - 6x + 5 = 0$

$2y = 6x - 5$

$y = 3x - \frac{5}{2}$

$3x^2 - 6(3x - 5/2) + 9 = 0$

$3(x^2 - 2(3x - 5/2) + 3) = 0$

$x^2 - 6x + 5 + 3 = 0$

$x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$

$x = 2,\ y = 3(2) - 5/2 = 7/2$

$x = 4,\ y = 3(4) - 5/2 = 19/2$

$f_{xx} = 6x$

$f_{yy} = 2$

$f_{xy} = -6$

$D = f_{xx} f_{yy} - (f_{xy})^2$

$= 12x - 36$

$D(2, 7/2) = 24 - 36 < 0$

$(2, 7/2)$ gives a saddle point.

$D(4, 19/2) = 48 - 36 > 0$.

$f_{xx}(4, 19/2) = 6(4) > 0$

$(4, 19/2)$ gives a minimum.

4. Suppose product A and product B are ***competitive***.

a) If the price of product A goes up, the demand for product A will go down.

b) If the price of product A goes up, the demand for product B will go Up.

c) Two products that might behave this way are Coke and Pepsi.

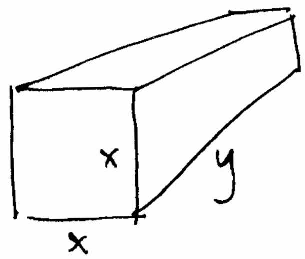
Suppose product A and product B are ***complementary***.

d) If the price of product A goes up, the demand for product A will go down.

e) If the price of product A goes up, the demand for product B will go down.

f) Two products that might behave this way are hot dogs and hot dog buns.

7. According to postal regulations, the girth (distance around) plus the length of parcels sent by 4th class mail may not exceed 108 inches. What is the largest possible volume of a rectangular parcel with two square sides that can be sent by 4th class mail?



To maximize volume,

$108 = 4x + y \quad \leftarrow$ constraint

$V = x^2 y \quad \leftarrow$ optimize this

$$F(x, y, \lambda) = f(x,y) - \lambda(g(x,y) - k)$$
$$= x^2 y - \lambda(4x + y - 108)$$

$F_x = 2xy - 4\lambda = 0 \longrightarrow 2xy - 4x^2 = 0$

$F_y = x^2 - \lambda = 0 \longrightarrow \lambda = x^2$

$2x(y - 2x) = 0$

$x = 0$ (no!)

or $y = 2x$

$F_\lambda = -4x - y + 108 = 0$

$$-4x - 2x + 108 = 0$$
$$-6x + 108 = 0$$
$$108 = 6x$$
$$x = \frac{108}{6} = 18 \text{ inches}$$
$$y = 2x = 36 \text{ inches}$$

Largest possible volume is

$V = x^2 y$

$V = (18)^2(36) = 10{,}368 \text{ in}^3$

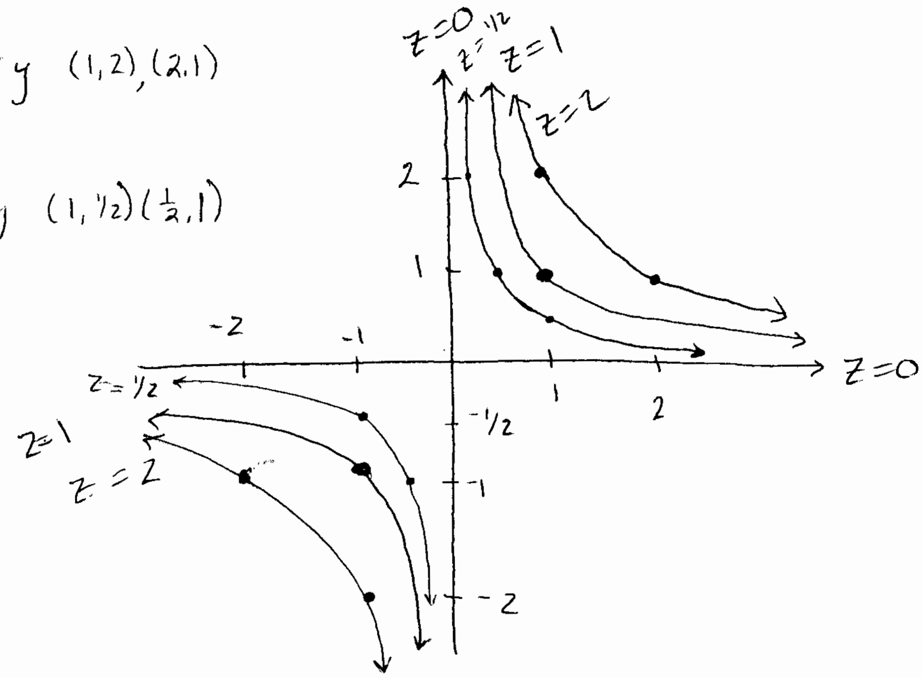
5. On a single plane, sketch and label 3 level curves of the surface $z = xy$.

$\underline{z=0}$: $x=0$ or $y=0$, curve is the coordinate axes.

$\underline{z=1}$: $1 = xy$ $(1,1), (-1,-1)$

$\underline{z=2}$: $2 = xy$ $(1,2), (2,1)$

$\underline{z=\frac{1}{2}}$: $\frac{1}{2} = xy$ $(1, \frac{1}{2}) (\frac{1}{2}, 1)$



6. Calculate $\int_1^\infty e^{1-x} dx$.

$$= \lim_{n\to\infty} \int_1^n e^{1-x}\,dx = \lim_{n\to\infty} \int_{x=1}^{x=n} e^u(-du)$$

$u = 1-x$

$du = -dx$

$-du = dx$

$$= \lim_{n\to\infty} \left[-\int_{x=1}^{x=n} e^u du \right]$$

$$= \lim_{n\to\infty} -\left[e^u \right]_{x=1}^{x=n}$$

$$= \lim_{n\to\infty} -\left[e^{1-x} \right]_1^n$$

$$= \lim_{n\to\infty} -\left[\underbrace{e^{1-n}}_{0} - e^0 \right] = -[0-1] = 1$$