Mathematics 204

Fall 2010

Final Exam

Your Printed Name: Dr. Grow

Your Instructor's Name: ____________________

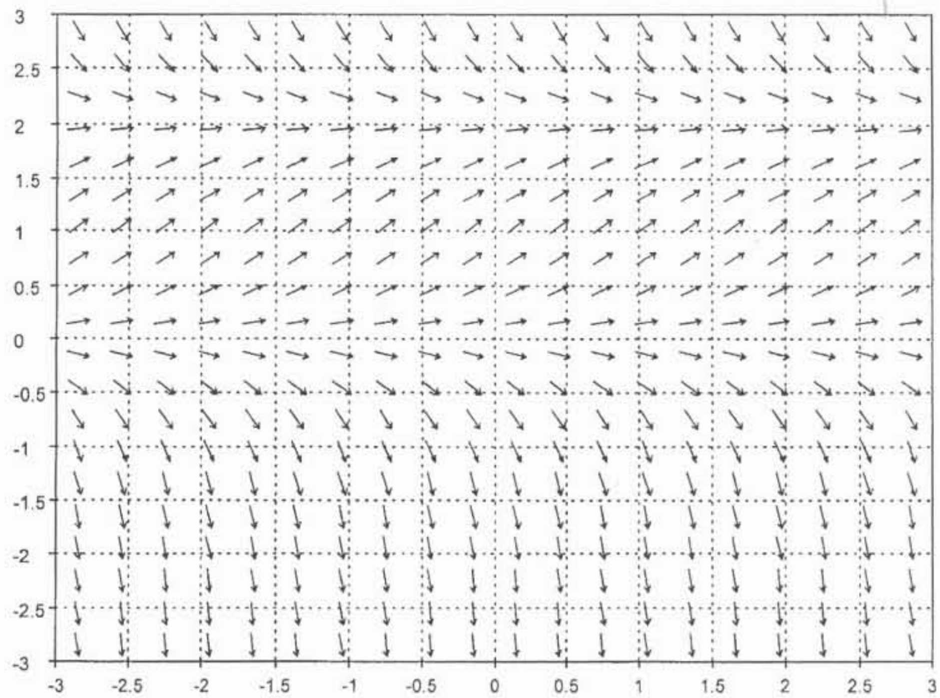
Your Section (or Class Meeting Days and Time): ____________________

1. **Do not open this exam until you are instructed to begin.**
2. All cell phones and other electronic noisemaking devices must be **turned off or completely silenced** (i.e. not on vibrate) for the duration of the exam.
3. The final exam consists of this cover page, 9 pages of problems containing 10 numbered problems, and a short table of Laplace transform formulas.
4. Once the exam begins, you will have 120 minutes to complete your solutions.
5. **Show all relevant work. No credit** will be awarded for unsupported answers and partial credit depends upon the work you show. In particular, all integrals, partial fraction decompositions, and matrix computations must be done by hand.
6. You may use the back of any page for extra scratch paper, but if you would like it to be graded, clearly indicate in the space of the original problem where the work is to be found.
7. The symbol [22] at the beginning of a problem indicates the point value of that problem is 22. The maximum possible score on this exam is 200.

problem	1	2	3	4	5	6	7	8	9	10	**Sum**
points earned											
maximum points	14	20	22	20	22	15	21	22	22	22	200

1.[14] Circle the letter corresponding to the differential equation whose direction field is given in the figure below. Then explain your reasons for your answer. Note that "My calculator says so." is not a valid reason.

(a) $y' = y+1$ (b) $y' = -y-1$ (c) $y' = y^2 - 2y$ (d) $y' = 2y - y^2$



The direction field corresponds to an autonomous differential equation with equilibrium solutions $y=0$ and $y=2$. Therefore the differential equations corresponding to (a) and (b) are eliminated because $y=-1$ is their only equilibrium solution. When $y=1$, (c) has $y' = 1^2 - 2(1) = -1$ while (d) has $y' = 2(1) - 1^2 = 1$. Consequently, since the direction field above shows a slope of approximately $+1$ when $y=1$, the differential equation (d) corresponds to the direction field above.

2.[20] Find the general solution of $y' = 2y + 4 - t$. (Linear, first-order DE)

$y' - 2y = 4 - t$ is the standard normalized form for the linear DE.

An integrating factor is

$$e^{\int p(t)dt} = e^{\int -2dt} = e^{-2t+\cancel{c}^{0}}.$$

Then $e^{-2t}(y' - 2y) = (4-t)e^{-2t}$ or

$$(*) \quad y'e^{-2t} - 2e^{-2t}y = (4-t)e^{-2t}.$$

Observe that the left member of the equation above is exact since the product rule implies $(ye^{-2t})' = y'e^{-2t} + (-2)e^{-2t}y$. Therefore $(*)$ is equivalent to

$$(ye^{-2t})' = (4-t)e^{-2t}.$$

Integrating both sides with respect to t yields

$$ye^{-2t} = \int (4-t)e^{-2t}dt.$$

Integrating by parts with $U = 4-t$ and $dV = e^{-2t}dt$ leads to

$$ye^{-2t} = (4-t)\frac{e^{-2t}}{-2} - \int \frac{e^{-2t}}{-2}(-dt)$$

$$= (-2 + \frac{t}{2})e^{-2t} + \frac{1}{4}e^{-2t} + c.$$

Multiplying throughout by e^{2t} yields the general solution:

$$y(t) = -2 + \frac{t}{2} + \frac{1}{4} + ce^{2t}$$

$$\boxed{y(t) = -\frac{7}{4} + \frac{t}{2} + ce^{2t}}$$

where c is an arbitrary constant.

3.[22] (a) According to Newton's law of cooling, the temperature of an object changes at a rate proportional to the difference between the temperature of the object and the ambient temperature (i.e. the temperature of the surroundings). Express Newton's law of cooling as a differential equation.

$$\boxed{\frac{dT}{dt} = k(T - T_0)}$$

Here $T(t)$ represents the temperature of the object at time t, T_0 is the ambient temperature, and k is a constant of proportionality.

(b) A murder victim was found on the street at 12:00 midnight and the temperature of the body at that time was 85°F. After 2 hours, the temperature of the body had dropped to 74°F. Between 9:00 PM and 2:00 AM, the outdoor temperature was steady at 68°F. Use Newton's law of cooling to determine the time the crime took place. You should assume the temperature of the body at the time of death was a standard 98.6°F, and be sure to give your answer in terms of a time of day as read on a clock - for example, 9:38 PM.

$$\frac{dT}{T - T_0} = k\,dt \quad \text{(Variables Separable)}$$

Integrating both sides gives

$$\ln(T - T_0) = kt + c.$$

Exponentiating yields

$$T - T_0 = e^{kt+c} = Ae^{kt} \quad (A = e^c)$$

so $T(t) = T_0 + Ae^{kt}$.

We will let $T(t)$ denote the Fahrenheit temperature t hours after 12:00 midnight. Then $T(0) = 85$ and $T(2) = 74$, so

$$85 = 68 + A$$

and

$$74 = 68 + Ae^{2k}.$$

Consequently $A = 17$ and $\frac{6}{17} = e^{2k}$

so $k = \frac{1}{2}\ln\left(\frac{6}{17}\right) \doteq -0.5207$.

Thus $T(t) = 68 + 17e^{\frac{t}{2}\ln\left(\frac{6}{17}\right)}$.

We need to find the time t when the temperature of the body was 98.6. Therefore

$$98.6 = 68 + 17e^{\frac{t}{2}\ln\left(\frac{6}{17}\right)}$$

so

$$\ln\left(\frac{98.6 - 68}{17}\right) = \frac{t}{2}\ln\left(\frac{6}{17}\right)$$

and

$$\frac{2\ln\left(\frac{98.6-68}{17}\right)}{\ln\left(\frac{6}{17}\right)} = t$$

or $t \doteq -1.129$. That is, the murder occurred 1 hour and 8 minutes (approximately) before midnight. This would be

$$\boxed{10:52 \text{ PM}}.$$

4.[20] (a) Solve $y''' - y = 0$. You may find the identity $a^3 - b^3 = (a-b)(a^2+ab+b^2)$ useful.

$y = e^{rt}$ in $y''' - y = 0$ leads to $r^3 - 1 = 0$. Using the identity with $a = r$ and $b = 1$ yields $(r-1)(r^2+r+1) = 0$. The solutions are $r = 1$ and $r = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$.

The general solution of $y''' - y = 0$ is

$$\boxed{y = c_1 e^{t} + c_2 e^{-t/2}\cos\left(\tfrac{\sqrt{3}}{2}t\right) + c_3 e^{-t/2}\sin\left(\tfrac{\sqrt{3}}{2}t\right)}$$

where c_1, c_2, and c_3 are arbitrary constants.

(b) Solve $t^2 y'' + 3ty' + y = 0$ on the interval $t > 0$.

$y = t^m$ in $t^2y'' + 3ty' + y = 0$ leads to $m(m-1) + 3m + 1 = 0$. Then $0 = m^2 + 2m + 1 = (m+1)^2$ so $m = -1$ (multiplicity two). The general solution of $t^2y'' + 3ty' + y = 0$ on $t > 0$ is

$$\boxed{y = c_1 t^{-1} + c_2 t^{-1}\ln(t)}$$

where c_1 and c_2 are arbitrary constants.

5.[22] Consider the differential equation $y'' - 2y' + y = \dfrac{e^t}{1+t^2}$.

(a) Classify the differential equation by giving its order, stating whether it is linear or nonlinear, homogeneous or nonhomogeneous, and whether it has constant or variable coefficients.

The DE is of order two, it is linear, nonhomogeneous, and has constant coefficients.

(b) Find the general solution of the differential equation.

$y = e^{rt}$ in $y'' - 2y' + y = 0$ leads to $r^2 - 2r + 1 = 0$ or $(r-1)^2 = 0$ so $r = 1$ with multiplicity two. It follows that $y_c(t) = c_1 e^t + c_2 te^t$ is the general solution of $y'' - 2y' + y = 0$ since $y_1(t) = e^t$, $y_2(t) = te^t$ form a fundamental set of solutions of $y'' - 2y' + y = 0$ because $W(y_1, y_2)(t) = \begin{vmatrix} e^t & te^t \\ e^t & (t+1)e^t \end{vmatrix} = (t+1)e^{2t} - te^{2t} = e^{2t} \neq 0$. The variation of parameters formula

$$y_p = u_1(t)y_1(t) + u_2(t)y_2(t)$$

gives a particular solution of $y'' - 2y' + y = \dfrac{e^t}{1+t^2}$, provided $u_1(t) = \displaystyle\int -\frac{f(t)y_2(t)}{W(y_1,y_2)(t)}\,dt$ and $u_2(t) = \displaystyle\int \frac{f(t)y_1(t)}{W(y_1,y_2)(t)}\,dt$. In our case, these formulas become

$$u_1(t) = \int \frac{-\frac{e^t}{1+t^2}\cdot te^t}{e^{2t}}\,dt = \int -\frac{t\,dt}{1+t^2} = -\frac{1}{2}\ln(1+t^2) + \cancel{C}^{\,0}$$

$$u_2(t) = \int \frac{\frac{e^t}{1+t^2}\cdot e^t}{e^{2t}}\,dt = \int \frac{dt}{1+t^2} = \text{Arctan}(t) + \cancel{C}^{\,0}.$$

Therefore $y_p(t) = -\frac{1}{2}\ln(1+t^2)e^t + \text{Arctan}(t)\,te^t$ is a particular solution. The general solution is $y = y_c + y_p$ or

$$\boxed{y(t) = c_1 e^t + c_2 te^t - \frac{e^t}{2}\ln(1+t^2) + te^t\,\text{Arctan}(t)}$$

where c_1 and c_2 are arbitrary constants.

6.[15] (Please use 32 feet per second per second as the acceleration of gravity in this problem.) A spring hangs vertically from a rigid support. A body weighing 16 pounds stretches the spring 4 inches when it is first attached. The body is in a medium with a damping constant of 2 pound-seconds per foot. Suppose the body is displaced 3 inches above the equilibrium position and then released. If gravity is the only external force that acts on the body, set up, **BUT DO NOT SOLVE**, an initial value problem describing the body's motion.

$$mu'' + \beta u' + ku = f(t)$$

$$\boxed{\begin{array}{c} \frac{1}{2}u'' + 2u' + 48u = 0, \\ u(0) = -\frac{1}{4},\ u'(0) = 0 \end{array}}$$

$mg = \text{weight}$ so $m = \frac{16}{g} = \frac{16}{32} = \frac{1}{2}$ slug.

$\beta = 2$ lb/ft/sec

$ku_0 = \text{weight}$ so $k = \frac{16\,\text{lb}}{u_0} = \frac{16\,\text{lb}}{\frac{1}{3}\text{ft}} = 48$ lb/ft.

$f(t) = 0$ (Since gravity has already been accounted for in the 4 in. stretch to the equilibrium position.)

Here $u = u(t)$ represents the vertical displacement of the body (in feet) from its equilibrium position t seconds after it was released.

7.[21] Use the Laplace transform to solve the integral equation $y(t) + 3\overbrace{\int_0^t e^{-4(t-\tau)} y(\tau)\,d\tau}^{\text{convolution product of } e^{-4t} \text{ and } y(t)} = 1$.

$$\mathcal{L}\{y(t) + 3e^{-4t} * y(t)\}(s) = \mathcal{L}\{1\}(s)$$

$$Y(s) + \frac{3}{s+4} \cdot Y(s) = \frac{1}{s}$$

$$Y(s)\left[1 + \frac{3}{s+4}\right] = \frac{1}{s}$$

$$Y(s)\left[\frac{s+7}{s+4}\right] = \frac{1}{s}$$

$$Y(s) = \frac{s+4}{s(s+7)}$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{s+4}{s(s+7)}\right\}$$

$$\frac{s+4}{s(s+7)} \overset{\text{P.F.D.}}{=} \frac{A}{s} + \frac{B}{s+7}$$

$$s+4 = A(s+7) + Bs$$

$s=0$: $4 = 7A \Rightarrow A = 4/7$

$s=-7$: $-3 = -7B \Rightarrow B = 3/7$

$$\therefore y(t) = \mathcal{L}^{-1}\left\{\frac{4/7}{s} + \frac{3/7}{s+7}\right\}$$

$$\boxed{y(t) = \frac{4}{7} + \frac{3}{7}e^{-7t}}$$

8.[22] Solve the initial value problem $y'' + 4y = f(t)$, $y(0) = 1$, $y'(0) = 0$, if $f(t) = \begin{cases} 0 & \text{if } t < 2, \\ -4 & \text{if } t \geq 2. \end{cases}$

Then compute the value of the solution, accurate to three decimal places, when $t = 1$ and $t = 3$.

$f(t) = -4u_2(t)$ so the DE becomes $y'' + 4y = -4u_2(t)$. Taking the Laplace transform of both sides of the DE yields

$$s^2 Y(s) - s\cancel{y(0)}^{1} - \cancel{y'(0)}^{0} + 4Y(s) = \frac{-4e^{-2s}}{s}$$

where $Y(s) = \mathcal{L}^{-1}\{y\}(s)$. Rearranging,

$$(s^2+4)Y(s) = s - \frac{4e^{-2s}}{s}$$

$$Y(s) = \frac{s}{s^2+4} - e^{-2s} \cdot \frac{4}{s(s^2+4)}.$$

Taking the inverse Laplace transform gives

$$y(t) = \mathcal{L}^{-1}\left\{ \frac{s}{s^2+4} - e^{-2s} \cdot \frac{4}{s(s^2+4)} \right\}$$

$$= \cos(2t) - u_2(t) f(t-2)$$

where $f(t) = \mathcal{L}^{-1}\left\{ \frac{4}{s(s^2+4)} \right\}$. A partial fraction decomposition shows

$\frac{4}{s(s^2+4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4}$ so $4 = A(s^2+4) + (Bs+C)s$. Setting $s=0$ gives $4 = 4A$ so $A = 1$. Setting $s = 2i$ gives $4 = -4B + 2iC$ so $B = -1$ and $C = 0$.

Therefore $f(t) = \mathcal{L}^{-1}\left\{ \frac{1}{s} - \frac{s}{s^2+4} \right\} = 1 - \cos(2t)$. Thus

$\boxed{y(t) = \cos(2t) - u_2(t)\left[1 - \cos(2(t-2))\right].}$ Then $y(1) = \cos(2) \doteq \boxed{-0.416}$

and $y(3) = \cos(6) - \left[1 - \cos(2)\right] \doteq \boxed{-.456}$

since $u_2(1) = 0$ and $u_2(3) = 1$.

9.[22] Find the general solution of the system $\mathbf{x}' = \begin{pmatrix} -6 & 4 \\ -1 & -2 \end{pmatrix}\mathbf{x}$ and describe the behavior of solutions as $t \to \infty$.

Let $A = \begin{bmatrix} -6 & 4 \\ -1 & -2 \end{bmatrix}$. Then $\vec{x} = \vec{k}e^{\lambda t}$ in $\vec{x}' = A\vec{x}$ leads to $\lambda\vec{k} = A\vec{k}$ or $(A-\lambda I)\vec{k} = \vec{0}$.

Nontrivial solutions $\vec{k}$ exist provided

$$0 = \det(A - \lambda I) = \begin{vmatrix} -6-\lambda & 4 \\ -1 & -2-\lambda \end{vmatrix} = (2+\lambda)(6+\lambda) + 4 = \lambda^2 + 8\lambda + 16 = (\lambda+4)^2.$$

Therefore $\lambda = -4$ is an eigenvalue of multiplicity two. An eigenvector $\vec{k}$ satisfies $(A-\lambda I)\vec{k} =$ or equivalently

$$\begin{bmatrix} -6+4 & 4 \\ -1 & -2+4 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{or} \quad \begin{cases} \cancel{-2k_1 + 4k_2 = 0} & \text{Redundant; equation 1 is twice equation 2.} \\ -k_1 + 2k_2 = 0 & \Rightarrow k_1 = 2k_2 \end{cases}$$

$\therefore \ \vec{k} = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 2k_2 \\ k_2 \end{bmatrix} = k_2\begin{bmatrix} 2 \\ 1 \end{bmatrix}$. We take $k_2 = 1$ so $\vec{k}^{(1)} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\vec{x}^{(1)} = \vec{k}^{(1)}e^{\lambda_1 t} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}e^{-4t}$

is a solution to $\vec{x}' = A\vec{x}$. To get a second solution that is not linearly dependent upon $\vec{x}^{(1)}$ we assume $\vec{x} = \vec{k}te^{\lambda t} + \vec{\ell}e^{\lambda t}$ Substituting in $\vec{x}' = A\vec{x}$ leads to $\begin{cases} (A-\lambda I)\vec{k} = \vec{0} \\ (A-\lambda I)\vec{\ell} = \vec{k}. \end{cases}$

We have already solved the first equation : $\lambda = -4$ and $\vec{k} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (up to a constant factor). To solve the second equation we must solve $\begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ or equivalently

$$\begin{cases} \cancel{-2\ell_1 + 4\ell_2 = 2} & \text{Redundant} \\ -\ell_1 + 2\ell_2 = 1 & \Rightarrow 2\ell_2 - 1 = \ell_1. \end{cases}$$

Thus $\vec{\ell} = \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix} = \begin{bmatrix} 2\ell_2 - 1 \\ \ell_2 \end{bmatrix} = \ell_2\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix}$. We set $\ell_2 = 0$ for convenience. Then

$$\vec{x}^{(2)} = \vec{k}te^{\lambda t} + \vec{\ell}e^{\lambda t} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}te^{-4t} + \begin{bmatrix} -1 \\ 0 \end{bmatrix}e^{-4t}$$

Since $W(\vec{x}^{(1)}\ \vec{x}^{(2)})(t) = \det\begin{bmatrix} 2e^{-4t} & 2te^{-4t} - 1 \\ e^{-4t} & te^{-4t} \end{bmatrix} = e^{-4t} \neq 0$, $\vec{x}^{(1)}$ and $\vec{x}^{(2)}$ form a fundamental set of solutions. The general solution of $\vec{x}' = A\vec{x}$ is

$$\boxed{\vec{x}(t) = c_1\begin{bmatrix} 2 \\ 1 \end{bmatrix}e^{-4t} + c_2\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}te^{-4t} + \begin{bmatrix} -1 \\ 0 \end{bmatrix}e^{-4t}\right)}$$

where c_1 and c_2 are arbitrary constants. Since $e^{-4t} \to 0$ and $te^{-4t} \to 0$ as $t \to \infty$, $\boxed{\vec{x}(t) \to \begin{bmatrix} 0 \\ 0 \end{bmatrix}}$ as $t \to \infty$.

10.[22] Given that $\mathbf{x}(t) = c_1\begin{pmatrix} e^{-t} \\ e^{-t} \end{pmatrix} + c_2\begin{pmatrix} 2e^{-2t} \\ 3e^{-2t} \end{pmatrix}$ is the general solution for the homogeneous system $\mathbf{x}' = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix}\mathbf{x}$, solve the nonhomogeneous initial value problem $\mathbf{x}' = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 2 \\ 2 \end{pmatrix}e^{-t}$, $\mathbf{x}(0) = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$.

$\Psi(t) = \begin{bmatrix} e^{-t} & 2e^{-2t} \\ e^{-t} & 3e^{-2t} \end{bmatrix}$ is a fundamental matrix for $\vec{x}' = A\vec{x}$ where $A = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$.

$\vec{x}_p = \Psi(t)\int_{t_0}^{t} \Psi^{-1}(s)\vec{g}(s)\,ds$ is a particular solution to $\vec{x}' = A\vec{x} + \vec{g}(t)$. In our case,

$\Psi^{-1}(s) = \dfrac{1}{\det \Psi(s)}\begin{bmatrix} \psi_{22}(s) & -\psi_{12}(s) \\ -\psi_{21}(s) & \psi_{11}(s) \end{bmatrix} = \dfrac{1}{e^{-3s}}\begin{bmatrix} 3e^{-2s} & -2e^{-2s} \\ -e^{-s} & e^{-s} \end{bmatrix} = \begin{bmatrix} 3e^{s} & -2e^{s} \\ -e^{2s} & e^{2s} \end{bmatrix}$ so

$\vec{x}_p = \Psi(t)\int_{t_0}^{t} \begin{bmatrix} 3e^{s} & -2e^{s} \\ -e^{2s} & e^{2s} \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix}e^{-s}\,ds = \Psi(t)\int_0^t \begin{bmatrix} 2e^{s} \\ 0 \end{bmatrix}e^{-s}\,ds = \Psi(t)\int_0^t \begin{bmatrix} 2 \\ 0 \end{bmatrix}ds$ and

$\vec{x}_p = \begin{bmatrix} e^{-t} & 2e^{-2t} \\ e^{-t} & 3e^{-2t} \end{bmatrix}\begin{bmatrix} 2t \\ 0 \end{bmatrix} = 2te^{-t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. The general solution of $\vec{x}' = A\vec{x} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}e^{-t}$ is

$\vec{x} = \vec{x}_c + \vec{x}_p = c_1\begin{bmatrix} 1 \\ 1 \end{bmatrix}e^{-t} + c_2\begin{bmatrix} 2 \\ 3 \end{bmatrix}e^{-2t} + 2te^{-t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ where c_1 and c_2 are arbitrary constants. We need to determine the constants so the initial condition is satisfied:

$\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \vec{x}(0) = c_1\begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ so $c_2 = 2$ and $c_1 = -1$. Thus

$$\boxed{\vec{x}(t) = -\begin{bmatrix} 1 \\ 1 \end{bmatrix}e^{-t} + 2\begin{bmatrix} 2 \\ 3 \end{bmatrix}e^{-2t} + 2te^{-t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

or

$$\boxed{\vec{x}(t) = \begin{bmatrix} (2t-1)e^{-t} + 4e^{-2t} \\ (2t-1)e^{-t} + 6e^{-2t} \end{bmatrix}}$$

solves the initial value problem.

A SHORT TABLE OF LAPLACE TRANSFORMS

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
1. 1	$\frac{1}{s}$
2. e^{at}	$\frac{1}{s-a}$
3. t^n	$\frac{n!}{s^{n+1}}, \quad n = 1, 2, 3 \ldots.$
4. $e^{at}t$	$\frac{1}{(s-a)^2}$
5. $e^{at}\sin(bt)$	$\frac{b}{(s-a)^2 + b^2}$
6. $e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2 + b^2}$
7. $e^{at}f(t)$	$F(s-a)$
8. $f'(t)$	$sF(s) - f(0)$
9. $f''(t)$	$s^2F(s) - sf(0) - f'(0)$
10. $\delta(t-a)$	e^{-as}
11. $u_a(t)$	$\frac{e^{-as}}{s}$
12. $u_a(t)f(t-a)$	$e^{-as}F(s)$
13. $\int_0^t f(\tau)g(t-\tau)d\tau$	$F(s)G(s)$

Math 204 Final Exam "Master List" Scorecard, 2010 Fall Semester

200
199 I
198 I
197 I
196 I
195 I
194 III
193
192 IIII
191 III
190 II
37 A's ✓
189 I
188 I
(10.0%)
187 III
186
185 I
184 III
183 II
182 III
181 IIII
180 II

---

179 II
178 II
177
176 卌
175 III
174 I
173 I
172 II
52 B's ✓
171 I
170 III
(14.1%)
169 I
168 II
167 卌
166 III
165 III
164 IIII
163 IIII
162 II
161 III
160 卌

---

159 III
158 III
157 II
156 II
155 I
154 IIII
153 IIII
152 IIII
151 III
150 I
149 卌
148 II
147 I
146 IIII
145
56 C's ✓
144 II
143 III
(15.2%)
142 IIII
141 III
140 卌

---

139 IIII
138 IIII
137 I
136 卌
135 I
134 II
133 I
132 III
131 卌
58 D's ✓
130 II
129 II
128 I
(15.7%)
127 II
126 卌
125 III
124 III
123 IIII
122 卌
121 III
120 II

---

119 I
118 卌
117 IIII
116 II
115 I
114 III
113 I
112 III
111 II
110 IIII
109 卌 I
166 F's ✓
108 卌
107 IIII
(45.0%)
106
105 II
104 II
103 I
102 卌 I
101
100 III
99 IIII
98 II
97 III
96 II
95 III
94 I
93 III
92 I
91 I
90 II
89
88 IIII
87 I
86 I
85 III
84 III
83
82 II
81 I
80 IIII
79
78 I
77 I
76 IIII
75 I
74 III
73 II
72 III
71 I
70 I
69 III
68 II
67 I
66 III
65 III
64 II
63 I
62 I
61
60 IIII
59
58 I
57 I
56 II
55 I
54
53 II
52 II
51
50
49 I
48 II
47 II
46 II
45 I
44 I
43 III
42 II
41 I
40
39
38 III
37
36
35
34
33
32
31
30
29 I
28
27 I
26
25 I
24 I
23
22
21
20
19
18
17
16
15 I
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0

Number taking final: 369 ✓
Median: 125
Mean: 122.68
Standard Deviation: 43.12