Math 315 Spring 2013 Midterm Exam Name:______________________________

1.(26 pts.) Define each the following terms, phrases, or symbols.

a. The set E is countable.

b. The subset E of $\mathbb{R}$ is of measure zero.

d. Define $U(P,f,\alpha)$, $L(P,f,\alpha)$, $\overline{\int_a^b} f d\alpha$, and $\underline{\int_a^b} f d\alpha$ where f is a bounded real function on $[a,b]$ and α is increasing on $[a,b]$.

e. The bounded, real function f is Riemann-Stieltjes integrable with respect to the increasing function α on $[a,b]$.

f. The real function α is of bounded variation on $[a,b]$.

g. Define $\int_a^b f d\alpha$ where f is continuous and α is of bounded variation on $[a,b]$.

h. The sequence of real functions $\langle f_n \rangle_{n=1}^{\infty}$ converges to f pointwise on the set E.

i. The sequence of real functions $\langle f_n \rangle_{n=1}^{\infty}$ converges to f uniformly on the set E.

j. The function N is a norm on a vector space X.

k. The sequence $\langle \mathbf{x}_n \rangle_{n=1}^{\infty}$ in a normed linear space (X,N) is Cauchy.

l. The sequence $\langle \mathbf{x}_n \rangle_{n=1}^{\infty}$ in a normed linear space (X,N) is convergent.

m. (X,N) is a Banach space.

2.(24 pts.) Give statements for each of the following.

a. State Lebesgue's theorem characterizing the set of points at which a monotonic function is differentiable.

b. State conditions under which the Riemann-Stieltjes integral of a function will exist.

c. State Holder's inequality and the conditions under which it holds.

d. State Jordan's theorem relating functions of bounded variation and monotonic functions.

e. State the integration by parts theorem for Riemann-Stieltjes integrals.

f. State the Weierstrass M-test.

3.(50 pts.) In each of the following cases, give an example or tell why such an example is impossible.

a. An uncountable subset of the real numbers which has measure zero.

b. A countable subset of the real numbers which is not of measure zero.

c. A function which is not Riemann-Stieltjes integrable with respect to the Heaviside unit step function on the interval $[-1,1]$.

d. A differentiable function which is not of bounded variation on $[0,1]$.

e. A function of bounded variation on $[0,1]$ with an uncountable number of discontinuities.

f. A pointwise convergent sequence of real functions which is not uniformly convergent.

g. A uniformly convergent sequence of continuous functions whose limit is not continuous.

h. A convergent sequence of Riemann integrable functions whose limit is not Riemann integrable.

i. A normed linear space.

j. A Cauchy sequence in $(C[0,1], \| \ \|_u)$ which is not convergent.

Math 315 Midterm Exam

Spring 2013

n : 21

mean : 77.4

median : 79

standard deviation : 15.7

Range	Graduate Letter Grade	Undergraduate Letter Grade	Frequency
85-100	A	A	10
70 - 84	B	B	4
65 - 69	C	B	0
50 - 64	C	C	6
0 - 49	F	D	1