Mathematics 325 Exam II Name: Dr. Grow

Fall 2010

1.(33 pts.) Solve $u_t - u_{xx} = 0$ in the upper half-plane $-\infty < x < \infty,\ 0 < t < \infty$, subject to the initial condition $u(x,0) = e^{-x^2}$ for $-\infty < x < \infty$.

A solution to $u_t - ku_{xx} = 0$ in the upper half-plane, satisfying $u(x,0) = \varphi(x)$ for 3 pts. to here all real x, is given by $u(x,t) = \frac{1}{\sqrt{4k\pi t}}\int_{-\infty}^{\infty} e^{-\frac{(x-y)^2}{4kt}}\varphi(y)\,dy$. Therefore, a solution to our problem is

[illegible] pts. to here.
$$u(x,t) = \frac{1}{\sqrt{4\pi t}}\int_{-\infty}^{\infty} e^{-\frac{(x-y)^2}{4t}}\cdot e^{-y^2}\,dy = \frac{1}{\sqrt{4\pi t}}\int_{-\infty}^{\infty} e^{-\frac{\left[y^2-2xy+x^2+4ty^2\right]}{4t}}\,dy$$

$$= \frac{e^{-\frac{x^2}{4t}}}{\sqrt{4\pi t}}\int_{-\infty}^{\infty} e^{\frac{-\left[(1+4t)y^2-2yx\right]}{4t}}\,dy \qquad (*)$$

7 pts. to here We need to complete the square in y in the exponent of the integrand. Thus,

$$\frac{-\left[(1+4t)y^2-2yx\right]}{4t} = \frac{-\left[\left(\sqrt{1+4t}\,y\right)^2 - 2\left(\frac{x}{\sqrt{1+4t}}\right)\left(\sqrt{1+4t}\,y\right) + \left(\frac{x}{\sqrt{1+4t}}\right)^2 - \left(\frac{x}{\sqrt{1+4t}}\right)^2\right]}{4t}$$

$$= \frac{-\left(\sqrt{1+4t}\,y - \frac{x}{\sqrt{1+4t}}\right)^2}{4t} + \frac{x^2}{4t(1+4t)}.$$

13 pts. to here.

Consequently, if we substitute this in (*), we obtain

23 pts. to here.
$$u(x,t) = \frac{e^{-\frac{x^2}{4t}}\cdot e^{\frac{x^2}{4t(1+4t)}}}{\sqrt{4\pi t}}\int_{-\infty}^{\infty} e^{\frac{-\left(\sqrt{1+4t}\,y - \frac{x}{\sqrt{1+4t}}\right)^2}{4t}}\,dy.$$ Let $p = \frac{\sqrt{1+4t}\,y - \frac{x}{\sqrt{1+4}}}{\sqrt{4t}}$

31 pts. to here. Then $dp = \sqrt{\frac{1+4t}{4t}}\,dy$, so $u(x,t) = \frac{e^{\frac{x^2}{4t}\left(\frac{1}{1+4t}-1\right)}}{\sqrt{4\pi t}}\int_{-\infty}^{\infty} e^{-p^2}\sqrt{\frac{4t}{1+4t}}\,dp =$

$\frac{e^{-\frac{x^2}{1+4t}}}{\sqrt{1+4t}}\cdot\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-p^2}\,dp$. But $\int_{-\infty}^{\infty} e^{-p^2}\,dp = \sqrt{\pi}$ so

33 pts. to here.
$$\boxed{u(x,t) = \frac{e^{-\frac{x^2}{1+4t}}}{\sqrt{1+4t}}}.$$

2.(34 pts.) Find a solution to the damped wave equation

(1) $$u_{tt} - u_{xx} + 2u_t = 0 \text{ in } 0 < x < \pi,\ 0 < t < \infty,$$

satisfying

(2)-(3) $$u_x(0,t) = 0 = u_x(\pi,t) \text{ for } t \geq 0,$$

(4) $$u(x,0) = 0 \text{ for } 0 \leq x \leq \pi,$$

and

(5) $$u_t(x,0) = \frac{1}{2} + \frac{1}{2}\cos(2x) \text{ for } 0 \leq x \leq \pi.$$

(You may find useful the facts that $y'' + 2y' + n^2 y = 0$ has general solution $y_0(t) = c_1 + c_2 e^{-2t}$ if $n = 0$, $y_1(t) = c_1 e^{-t} + c_2 t e^{-t}$ if $n = 1$, and $y_n(t) = e^{-t}\left(c_1 \cos\left(t\sqrt{n^2-1}\right) + c_2 \sin\left(t\sqrt{n^2-1}\right)\right)$ if $n = 2,3,4,\ldots$)

Bonus (10 pts.): Show that the solution to (1)-(2)-(3)-(4)-(5) is unique.

We use the method of separation of variables. We seek nontrivial solutions of the form $u(x,t) = X(x)T(t)$ to the homogeneous portion of the problem: (1)-(2)-(3)-(4). Substituting in (1) yields

2 pts to here

$$X(x)T''(t) - X''(x)T(t) + 2X(x)T'(t) = 0$$

or $$-\frac{X''(x)}{X(x)} = \frac{-(T''(t) + 2T'(t))}{T(t)} = \text{constant} = \lambda .$$

[illegible] pts. to here

Substituting in (2) and (3) gives $X'(0)T(t) = 0 = X'(\pi)T(t)$ for $t \geq 0$, while substituting in (4) yields $X(x)T(0) = 0$ for $0 \leq x \leq \pi$. In order that $u(x,t) = X(x)T(t)$ is not zero, we must have $X'(0) = 0 = X'(\pi)$ and $T(0) = 0$. Thus we have the coupled system of ODE's and B.C's:

6 pts to here.

$$\left\{ \begin{array}{l} \boxed{X''(x) + \lambda X(x) = 0,\ X'(0) = 0 = X'(\pi)} \longleftarrow \text{Eigenvalue Problem} \\ T''(t) + 2T'(t) + \lambda T(t) = 0,\ T(0) = 0. \end{array} \right.$$

3 pts. to here.

By Sec. 4.2, the eigenvalues and eigenfunctions of the eigenvalue problem (in the presence of Neumann boundary conditions) are $\lambda_n = \left(\frac{n\pi}{l}\right)^2 = \left(\frac{n\pi}{\pi}\right)^2 = n^2$ and $X_n(x) = \cos\left(\frac{n\pi x}{l}\right) = \cos\left(\frac{n\pi x}{\pi}\right) = \cos(nx)$ where $n = 0, 1, 2, 3, \ldots$ Substituting $\lambda = \lambda_n = n^2$ in the T-equations produces $T_n''(t) + 2T_n'(t) + n^2 T_n(t) = 0$, $T_n(0) = 0$. By the "useful facts" (see above), the general solution of the T-differential equation is:

11 pts to here

14 pts. to here

16 pts to here

$$T_0(t) = c_1 + c_2 e^{-2t} \text{ if } n = 0,\ T_1(t) = c_1 e^{-t} + c_2 t e^{-t} \text{ if } n = 1, \text{ and}$$

$$T_n(t) = e^{-t}\left[c_1 \cos(t\sqrt{n^2-1}) + c_2 \sin(t\sqrt{n^2-1})\right] \text{ if } n = 2, 3, 4, \ldots$$

1 s. to here

We apply the boundary condition $T_n'(0)=0$ and find that, up to a constant factor,

22 pts. to here. $T_0(t) = 1 - e^{-2t}$, $T_1(t) = te^{-t}$, and $T_n(t) = e^{-t}\sin\left(t\sqrt{n^2-1}\right)$ if $n=2,3,4,\ldots$

Applying the superposition principle to the homogeneous problem (1)-(2)-(3)-(4), we see that, for any integer $N \geq 0$ and any choice of constants $A_0, A_1, \ldots, A_N$,

25 pts. to here

$$u(x,t) = \sum_{n=0}^{N} A_n X_n(x) T_n(t) = A_0(1-e^{-2t}) + A_1\cos(x)te^{-t} + \sum_{n=2}^{N} A_n\cos(nx)e^{-t}\sin(t\sqrt{n^2-}$$

solves (1)-(2)-(3)-(4). Then

27 pts. to here.

$$u_t(x,t) = 2A_0e^{-2t} + A_1\cos(x)(1-t)e^{-t} + \sum_{n=2}^{N} A_n\cos(nx)e^{-t}\left[\sqrt{n^2-1}\cos(t\sqrt{n^2-1}) - \sin(t\sqrt{n^2-1})\right],$$

so to satisfy (5) we must have

29 pts to here.

$$\frac{1}{2} + \frac{1}{2}\cos(2x) = u_t(x,0) = 2A_0 + A_1\cos(x) + \sum_{n=2}^{N} A_n\sqrt{n^2-1}\cos(nx) \qquad \text{for all } 0 \leq x \leq \pi.$$

32 pts. to here. By inspecting coefficients, we have $\frac{1}{2} = 2A_0$, $\frac{1}{2} = A_2\sqrt{2^2-1}$, and all other $A_n = 0$.

Thus,

34 pts to here

$$\boxed{u(x,t) = \frac{1}{4}(1-e^{-2t}) + \frac{1}{2\sqrt{3}}\cos(2x)e^{-t}\sin(t\sqrt{3})}$$

is a solution to (1)-(2)-(3)-(4)-(5).

<u>Bonus</u>: Let $u = v(x,t)$ be any solution to (1)-(2)-(3) and consider its energy function

1 pt. to here $E(t) = \int_0^{\pi}\left[\frac{1}{2}v_t^2(x,t) + \frac{1}{2}v_x^2(x,t)\right]dx$ for $0 \leq t < \infty$. Then

$$E'(t) = \int_0^{\pi}\frac{\partial}{\partial t}\left[\frac{1}{2}v_t^2(x,t) + \frac{1}{2}v_x^2(x,t)\right]dx$$

2 pts to here.

$$= \int_0^{\pi}\left[v_t(x,t)v_{tt}(x,t) + \overbrace{v_x(x,t)}^{U}\overbrace{v_{xt}(x,t)}^{dV}\right]dx \longleftarrow \text{(integrate by parts on the second integrand term)}$$

pts to here

$$= \int_0^{\pi} v_t(x,t)v_{tt}(x,t)dx + \underbrace{\cancel{v_x(x,t)v_t(x,t)}\Big|_{x=0}^{\pi}}_{0 \text{ by (2)-(3)}} - \int_0^{\pi} v_t(x,t)v_{xx}(x,t)dx$$

(OVER)

4 pts to here So $E'(t) = \int_0^\pi v_t(x,t)\overbrace{\left[v_{tt}(x,t) - v_{xx}(x,t)\right]}^{\text{Apply (1).}}dx = -2\int_0^\pi \left[v_t(x,t)\right]^2 dx \leq 0.$

5 pts to here That is, $E = E(t)$ is a decreasing function on $t \geq 0$. Now suppose that $u = w(x,t)$ is another solution to (1)-(2)-(3)-(4)-(5) and consider

6 pts. to here

$$v(x,t) = u(x,t) - w(x,t) \qquad \text{for } 0 \leq x \leq \pi,\ 0 \leq t < \infty,$$

where $u = u(x,t)$ is the solution to (1)-(2)-(3)-(4)-(5) we obtained earlier in this problem. Then v solves

$$(1') \qquad v_{tt} - v_{xx} + 2v_t = 0 \quad \text{in } 0 < x < \pi,\ 0 < t < \infty,$$

$$(2')\text{-}(3') \qquad v_x(0,t) = 0 = v_x(\pi,t) \quad \text{for } t \geq 0,$$

7 pts. to here

$$(4')\text{-}(5') \qquad v(x,0) = 0 = v_t(x,0) \quad \text{for } 0 \leq x \leq \pi.$$

By the preceding work, the energy function $E(t) = \int_0^\pi \left[\frac{1}{2}v_t^2(x,t) + \frac{1}{2}v_x^2(x,t)\right]dx$ for v on $t \geq 0$ is decreasing. Therefore, for $t \geq 0$,

8 pts to here

$$0 \leq E(t) \leq E(0) = \int_0^\pi \left[\frac{1}{2}v_t^2(x,0) + \frac{1}{2}v_x^2(x,0)\right]dx = 0$$

by (5') and differentiation of (4'). The vanishing theorem then guarantees that $\frac{1}{2}v_t^2(x,t) + \frac{1}{2}v_x^2(x,t) = 0$ for $0 \leq x \leq \pi$ and $0 \leq t < \infty$. Therefore,

pts. to here. $v_t(x,t) = 0 = v_x(x,t)$ for $0 \leq x \leq \pi$ and $0 \leq t < \infty$ so $v(x,t) = \text{constant}$ in the strip $0 \leq x \leq \pi,\ 0 \leq t < \infty$. But (4') implies $v(x,0) = 0$ for $0 \leq x \leq \pi$

10 pts to here so $v(x,t) = 0$ in the strip. That is, $u(x,t) = w(x,t)$ for all $0 \leq x \leq \pi,\ 0 \leq t < \infty$ thus showing that there is only one solution to (1)-(2)-(3)-(4)-(5).

3.(33 pts.) Use Fourier transform methods to derive the formula

$$u(x,t)=\frac{1}{2}\int_{0}^{t}\int_{x-(t-\tau)}^{x+t-\tau} f(s,\tau)\,ds\,d\tau$$

for a solution to the inhomogeneous wave equation in the xt – plane,

$$① \qquad u_{tt}-u_{xx}=f(x,t),$$

satisfying homogeneous initial conditions: $u(x,0)\overset{②}{=}0\overset{③}{=}u_t(x,0)$ for $-\infty<x<\infty$.

Bonus (10 pts.) Compute a solution to $u_{tt}-u_{xx}=xt$ in the xt – plane which satisfies homogeneous initial conditions.

Let $u=u(x,t)$ be a solution to ① for $-\infty<x<\infty$, $-\infty<t<\infty$, satisfying ②-③ for $-\infty<x<\infty$. We take the Fourier transform of both sides of (1) with respect to x to obtain

1 pt. to here. $$\mathcal{F}(u_{tt})(\xi)-\mathcal{F}(u_{xx})(\xi)=\hat{f}(\xi,t),$$

2 pts. to here $$\frac{\partial^2}{\partial t^2}\mathcal{F}(u)(\xi)-(i\xi)^2\mathcal{F}(u)(\xi)=\hat{f}(\xi,t),$$

3 pts. to here (*) $$\frac{\partial^2}{\partial t^2}\mathcal{F}(u)(\xi)+\xi^2\mathcal{F}(u)(\xi)=\hat{f}(\xi,t).$$

The general solution of the second-order $\overset{(*)}{\text{ODE}}$ in t (with parameter ξ) is

4 pts. to here. $$\mathcal{F}(u)(\xi)=U_h(\xi,t)+U_p(\xi,t)$$

6 pts. to here where $U_h(\xi,t)=c_1(\xi)\cos(\xi t)+c_2(\xi)\sin(\xi t)$ is the general solution of the associated homogeneous equation of (*), $\frac{\partial^2}{\partial t^2}\mathcal{F}(u)(\xi)+\xi^2\mathcal{F}(u)(\xi)=0$, and $U_p(\xi,t)$ is a particular solution of (*). We use variation of parameters,

7 pts. to here. $$U_p(\xi,t)=u_1(\xi,t)\cos(\xi t)+u_2(\xi,t)\sin(\xi t),$$

to find a particular solution of (*). Here

8 pts. to here. $$u_1(\xi,t)=\int_0^t -\frac{Fy_2}{W}\,d\tau \quad \text{and} \quad u_2(\xi,t)=\int_0^t \frac{Fy_1}{W}\,d\tau$$

9 pts. to here where $y_1=\cos(\xi t)$, $y_2=\sin(\xi t)$ form a fundamental set of solutions

(OVER)

[illegible] pts. to here of the associated homogeneous equation of (*), $F = \hat{f}(\xi, t)$ is the forcing function of (*), and

12 pts. to here

$$W = W(y_1, y_2)(\xi) = \begin{vmatrix} y_1(\xi,t) & y_2(\xi,t) \\ \frac{\partial}{\partial t} y_1(\xi,t) & \frac{\partial}{\partial t} y_2(\xi,t) \end{vmatrix} = \begin{vmatrix} \cos(\xi t) & \sin(\xi t) \\ -\xi \sin(\xi t) & \xi\cos(\xi t) \end{vmatrix} = \xi$$

is the Wronskian of y_1, y_2. Therefore

13 pts. to here.

$$\mathcal{F}(u)(\xi) = c_1(\xi)\cos(\xi t) + c_2(\xi)\sin(\xi t) + \cos(\xi t)\int_0^t -\frac{\hat{f}(\xi,\tau)\sin(\xi\tau)}{\xi}\,d\tau + \sin(\xi t)\int_0^t \frac{\hat{f}(\xi,\tau)\cos(\xi\tau)}{\xi}\,d\tau$$

We use ②-③ to identify $c_1(\xi)$ and $c_2(\xi)$. First, ② implies

14 pts. to here.

$$0 = \mathcal{F}(u(\cdot,0))(\xi) = c_1(\xi) + 0 + 0 + 0 = c_1(\xi).$$

Next, observe that

$$\frac{\partial}{\partial t}\mathcal{F}(u)(\xi) = -\xi c_1(\xi)\sin(\xi t) + \xi c_2(\xi)\cos(\xi t) + -\xi\sin(\xi t)\int_0^t -\frac{\hat{f}(\xi,\tau)\sin(\xi\tau)}{\xi}\,d\tau$$

$$+ \cos(\xi t)\left[-\frac{\hat{f}(\xi,t)\sin(\xi t)}{\xi}\right] + \xi\cos(\xi t)\int_0^t \frac{\hat{f}(\xi,\tau)\cos(\xi\tau)}{\xi}\,d\tau$$

15 pts. to here.

$$+ \sin(\xi t)\left[\frac{\hat{f}(\xi,t)\cos(\xi t)}{\xi}\right].$$

By ③ we have

$$0 = \mathcal{F}(u_t(\cdot,0))(\xi) = \frac{\partial}{\partial t}\mathcal{F}(u)(\xi)\Big|_{t=0} = 0 + \xi c_2(\xi) + 0 + 0 + 0 + 0$$

16 pts. to here.

so $c_2(\xi) = 0$. Thus

[illegible] pts. to here.

$$\mathcal{F}(u)(\xi) = \cos(\xi t)\int_0^t -\frac{\hat{f}(\xi,\tau)\sin(\xi\tau)}{\xi}\,d\tau + \sin(\xi t)\int_0^t \frac{\hat{f}(\xi,\tau)\cos(\xi\tau)}{\xi}\,d\tau$$

$$= \int_0^t \frac{\hat{f}(\xi,\tau)}{\xi}\left[\sin(\xi t)\cos(\xi\tau) - \cos(\xi t)\sin(\xi\tau)\right]d\tau$$

[illegible] pts. to here.

$$= \int_0^t \frac{\hat{f}(\xi,\tau)\sin(\xi(t-\tau))}{\xi}\,d\tau\,. \qquad (\dagger)$$

Using formula A in the table of Fourier transforms with $b = t-\tau$, we see that $\frac{\sin(\xi(t-\tau))}{\xi} = \sqrt{\frac{\pi}{2}}\,\mathcal{F}\left(\chi_{(-(t-\tau),\,t-\tau)}\right)(\xi)$, so substituting in $(\dagger)$,

20 pts. to here.

$$\mathcal{F}(u)(\xi) = \int_0^t \mathcal{F}\left(f(\cdot,\tau)\right)(\xi)\sqrt{\frac{\pi}{2}}\,\mathcal{F}\left(\chi_{(-(t-\tau),\,t-\tau)}\right)(\xi)\,d\tau$$

22 pts. to here

$$= \int_0^t \frac{1}{\sqrt{2\pi}}\cdot\sqrt{\frac{\pi}{2}}\,\mathcal{F}\left(f(\cdot,\tau)*\chi_{(-(t-\tau),\,t-\tau)}\right)(\xi)\,d\tau$$

where we have used the convolution property $\frac{1}{\sqrt{2\pi}}\widehat{g*h}(\xi) = \hat{g}(\xi)\hat{h}(\xi)$. But interchanging the order of integration yields

$$\mathcal{F}(u)(\xi) = \frac{1}{2}\int_0^t \mathcal{F}\left(f(\cdot,\tau)*\chi_{(-(t-\tau),\,t-\tau)}\right)(\xi)\,d\tau$$

$$= \frac{1}{2}\int_0^t \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left(f(\cdot,\tau)*\chi_{(-(t-\tau),\,t-\tau)}\right)(x)e^{-i\xi x}\,dx\,d\tau$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left(\frac{1}{2}\int_0^t\left[f(\cdot,\tau)*\chi_{(-(t-\tau),\,t-\tau)}\right](x)\,d\tau\right)e^{-i\xi x}\,dx$$

24 pts. to here.

$$= \mathcal{F}\left(\frac{1}{2}\int_0^t\left[f(\cdot,\tau)*\chi_{(-(t-\tau),\,t-\tau)}\right](\cdot)\,d\tau\right)(\xi)\,.$$

By the uniqueness theorem for Fourier transforms, it follows that for all real x and t,

$$u(x,t) = \frac{1}{2}\int_0^t\left[f(\cdot,\tau)*\chi_{(-(t-\tau),\,t-\tau)}\right](x)\,d\tau$$

26 pts. to here.

$$= \frac{1}{2}\int_0^t\int_{-\infty}^{\infty} f(s,\tau)\,\chi_{(-(t-\tau),\,t-\tau)}(x-s)\,ds\,d\tau\,. \qquad (\square)$$

(OVER)

But $$\chi_{(-(t-\tau),\, t-\tau)}(x-s) = \begin{cases} 1 & \text{if} \quad -(t-\tau) < x-s < t-\tau, \\ 0 & \text{otherwise}, \end{cases}$$

$$= \begin{cases} 1 & \text{if} \quad x-(t-\tau) < s < x+t-\tau, \\ 0 & \text{otherwise}, \end{cases}$$

24 pts. to here. $$= \chi_{(x-(t-\tau),\, x+t-\tau)}(s) \qquad (\star)$$

for all real x and s, and all $0 < \tau < t$. Consequently, (□) and ($\star$) imply

31 pts. to here $$u(x,t) = \frac{1}{2}\int_0^t \int_{-\infty}^{\infty} f(s,\tau)\,\chi_{(x-(t-\tau),\, x+t-\tau)}(s)\,ds\,d\tau$$

35 pts. to here. $$= \boxed{\frac{1}{2}\int_0^t \int_{x-(t-\tau)}^{x+t-\tau} f(s,\tau)\,ds\,d\tau}\ .$$

1 pt. to here. **Bonus**: Applying the formula above with $f(x,t) = xt$ yields a solution to $u_{tt} - u_{xx} = xt$ in the xt-plane, subject to homogeneous initial conditions $u(x,0) = 0 = u_t(x,0)$ for $-\infty < x < \infty$. Therefore

2 pts. to here. $$u(x,t) = \frac{1}{2}\int_0^t \int_{x-(t-\tau)}^{x+t-\tau} s\tau\,ds\,d\tau = \frac{1}{2}\int_0^t \left.\frac{\tau s^2}{2}\right|_{s=x-(t-\tau)}^{x+t-\tau} d\tau = \frac{1}{4}\int_0^t \tau\left[(x+t-\tau)^2 - (x-(t-\tau))^2\right]d\tau$$

4 pts. to here. 5 pts. to here.

$$= \frac{1}{4}\int_0^t \tau 4x(t-\tau)\,d\tau = \left.\frac{1}{2}\tau^2 xt - \frac{1}{3}\tau^3 x\right|_{\tau=0}^{t} = \boxed{\frac{xt^3}{6}}\ .$$

7 pts. to here. 9 pts. to here. 10 pts. to here.

A Brief Table of Fourier Transforms

	$f(x)$	$\hat{f}(\xi)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-i\xi x}dx$
A.	$\begin{cases} 1 & \text{if } -b<x<b, \\ 0 & \text{otherwise.} \end{cases}$	$\sqrt{\frac{2}{\pi}}\frac{\sin(b\xi)}{\xi}$
B.	$\begin{cases} 1 & \text{if } c<x<d, \\ 0 & \text{otherwise.} \end{cases}$	$\frac{e^{-ic\xi}-e^{-id\xi}}{i\xi\sqrt{2\pi}}$
C.	$\frac{1}{x^2+a^2} \quad (a>0)$	$\sqrt{\frac{\pi}{2}}\frac{e^{-a\lvert\xi\rvert}}{a}$
D.	$\begin{cases} x & \text{if } 0<x\le b, \\ 2b-x & \text{if } b<x<2b, \\ 0 & \text{otherwise.} \end{cases}$	$\frac{-1+2e^{-ib\xi}-e^{-2ib\xi}}{\xi^2\sqrt{2\pi}}$
E.	$\begin{cases} e^{-ax} & \text{if } x>0, \\ 0 & \text{otherwise.} \end{cases}$	$\frac{1}{(a+i\xi)\sqrt{2\pi}}$
F.	$\begin{cases} e^{ax} & \text{if } b<x<c, \\ 0 & \text{otherwise.} \end{cases}$	$\frac{e^{(a-i\xi)c}-e^{(a-i\xi)b}}{(a-i\xi)\sqrt{2\pi}}$
G.	$\begin{cases} e^{iax} & \text{if } -b<x<b, \\ 0 & \text{otherwise.} \end{cases}$	$\sqrt{\frac{2}{\pi}}\frac{\sin(b(\xi-a))}{\xi-a}$
H.	$\begin{cases} e^{iax} & \text{if } c<x<d, \\ 0 & \text{otherwise.} \end{cases}$	$\frac{e^{ic(a-\xi)}-e^{id(a-\xi)}}{i(\xi-a)\sqrt{2\pi}}$
I.	$e^{-ax^2} \quad (a>0)$	$\frac{1}{\sqrt{2a}}e^{-\xi^2/(4a)}$
J.	$\frac{\sin(ax)}{x} \quad (a>0)$	$\begin{cases} 0 & \text{if } \lvert\xi\rvert\ge a, \\ \sqrt{\frac{\pi}{2}} & \text{if } \lvert\xi\rvert<a. \end{cases}$

Math 325

Exam II

Fall 2010

mean: 72.2

Standard deviation: 16.1

number taking exam: 37

	Distribution of Scores:	Graduate Letter Grade	Undergraduate Letter Grade	Frequency
𝍸 𝍸	87 – 100	A	A	10
𝍸 |||	73 – 86	B	B	8
𝍸 𝍸 |	60 – 72	C	B	11
𝍸	50 – 59	C	C	5
|||	0 – 49	F	D	3