Mathematics 325 Exam III Summer 2011 Name: Dr. Grow

**Note:** The last page of this exam contains statements of theorems for convergence of Fourier series.

1.(33 pts.) Consider the operator $T = -\dfrac{d^4}{dx^4}$ on the space

$$V = \left\{\varphi \in C^4[0,1]:\ \varphi'(0) = 0 = \varphi'(1) \text{ and } \varphi'''(0) = 0 = \varphi'''(1)\right\}.$$

(a) Show that $T$ is a symmetric operator on $V$, equipped with the usual inner product: $\langle \varphi, \psi \rangle = \int_0^1 \varphi(x)\overline{\psi(x)}dx$.

3 (b) Are all the eigenvalues of $T$ on $V$ real numbers? Justify your answer.

3 (c) Are all the eigenfunctions of $T$ on $V$ corresponding to distinct eigenvalues orthogonal on $[0,1]$? Why?

8 (d) Are all the eigenvalues of $T$ on $V$ nonpositive? Justify your answer.

11 (e) Compute all the eigenvalues and eigenfunctions for the operator $T$ on $V$. (Hint: You may find useful the fact that the general solution of $\varphi^{(4)}(x) - \alpha^4\varphi(x) = 0$ is

$$\varphi(x) = c_1\cos(\alpha x) + c_2\sin(\alpha x) + c_3\cosh(\alpha x) + c_4\sinh(\alpha x)$$

if $\alpha > 0$.)

8 (a) Let $f$ and $g$ belong to $V$. Then $\langle Tf, g\rangle = \int_0^1 -f^{(4)}(x)\overline{g(x)}\,dx \overset{\text{4 integrations by parts}}{=} \left(-f^{(3)}\overline{g} + f^{(2)}\overline{g'} - f'\overline{g''} + f\overline{g'''}\right)\Big|_0^1 - \int_0^1 f(x)\overline{g^{(4)}(x)}\,dx$. (4 pts to here.) But $f^{(3)}(1) = 0 = f^{(3)}(0)$, $g'(0) = 0 = g'(1)$, $f'(1) = 0 = f'(0)$, and $g'''(1) = 0 = g'''(0)$ so the boundary terms all evaluate to zero. (7 pts. to here.) Hence $\langle Tf, g\rangle = -\int_0^1 f(x)\overline{g^{(4)}(x)}\,dx = \langle f, Tg\rangle$. Therefore $T$ is a symmetric operator on $V$.

3 (b) $\boxed{\text{Yes}}$; (1 pt.) Theorem 2 in the lecture notes for Sec. 5.3 guarantees that eigenvalues for a symmetric operator are real numbers. (2 pts.)

3 (c) $\boxed{\text{Yes}}$; (1 pt.) Theorem 1 in the lecture notes for Sec. 5.3 guarantees that eigenfunctions of a symmetric operator that correspond to distinct eigenvalues are orthogonal. (2 pts.)

8 (d) $\boxed{\text{Yes, eigenvalues of } T \text{ on } V \text{ are nonpositive.}}$ (1 pt.) To see this suppose that $\lambda$ is an eigenvalue of $T$ on $V$ and let $\varphi$ be a nonzero function in $V$ such that $T\varphi = \lambda\varphi$. Without loss of generality, we may assume that $\varphi$ is real-valued. (Otherwise we may replace $\varphi$ by one of the functions $Re(\varphi) = \frac{1}{2}(\varphi + \bar{\varphi}) \in V$ or $Im(\varphi) = \frac{1}{2i}(\varphi - \bar{\varphi}) \in V$.) Then

5, (…o here.) $\lambda\langle\varphi,\varphi\rangle = \langle\lambda\varphi,\varphi\rangle = \langle T\varphi,\varphi\rangle = \int_0^1 -\varphi^{(4)}(x)\varphi(x)\,dx \overset{\text{two integrations by parts}}{=} \left(-\varphi^{(3)}\varphi + \varphi''\varphi'\right)\Big|_0^1 - \int_0^1 \varphi''(x)\varphi''(x)\,dx$

But $\varphi^{(3)}(1) = 0 = \varphi^{(3)}(0)$ and $\varphi'(1) = 0 = \varphi'(0)$ so the boundary terms all evaluate to zero.

(7pts. to here)

Hence $\lambda \overbrace{\langle \varphi, \varphi \rangle}^{\text{positive}} = -\int_0^1 [\varphi''(x)]^2 dx \leq 0$ so $\lambda \leq 0$. (3 pts. to here)

(1) (e) According to parts (b) and (d), the eigenvalues $\lambda$ of $T$ on $\mathbb{V}$ are real and nonpositive. (1 pt.)

(3) Case $\lambda = 0$: Then the eigenvalue condition $T\varphi = \lambda\varphi$ becomes $-\varphi^{(4)} = 0$ so $\varphi(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$. (1 pt. to here) Note that $\varphi'''(x) = 6c_3$ so $0 = \varphi'''(0)$ implies $c_3 = 0$. Also $\varphi'(x) = c_1 + 2c_2 x + 3c_3 x^2 = c_1 + 2c_2 x$ so $\varphi'(0) = 0 = \varphi'(1)$ implies $c_1 = c_2 = 0$. No constraints are placed on $c_0$ however, so $\boxed{\lambda = 0 \text{ is an eigenvalue and } \varphi_0(x) = 1 \text{ is a corresponding eigenfunction}}$ of $T$ on $\mathbb{V}$. (1 pt.) (1 pt.)

(7) Case $\lambda < 0$, say $\lambda = -\alpha^4$ where $\alpha > 0$: The eigenvalue condition $T\varphi = \lambda\varphi$ becomes $\varphi^{(4)} - \alpha^4\varphi = 0$ (1 pt.) The general solution of this differential equation is $\varphi(x) = c_1\cos(\alpha x) + c_2\sin(\alpha x) + c_3\cosh(\alpha x) + c_4\sinh(\alpha x)$. Observe that $\varphi'(x) = -\alpha c_1\sin(\alpha x) + \alpha c_2\cos(\alpha x) + \alpha c_3\sinh(\alpha x) + \alpha c_4\cosh(\alpha x)$ and $\varphi'''(x) = \alpha^3 c_1\sin(\alpha x) - \alpha^3 c_2\cos(\alpha x) + \alpha^3 c_3\sinh(\alpha x) + \alpha^3 c_4\cosh(\alpha x)$. Since $\varphi \in \mathbb{V}$, $\varphi'(1) = 0 = \varphi'(0)$ and $\varphi'''(1) = 0 = \varphi'''(0)$. Then $0 = \varphi'''(0) = -\alpha^3 c_2 + \alpha^3 c_4$ and $0 = \varphi'(0) = \alpha c_2 + \alpha c_4$, and it follows that $c_2 = 0 = c_4$. (2 pts.) Then $0 = \varphi'''(1) = \alpha^3 c_1 \sin(\alpha) + \alpha^3 c_3\sinh(\alpha)$ and $0 = \varphi'(1) = -\alpha c_1\sin(\alpha) + \alpha c_3\sinh(\alpha)$, or equivalently, $0 = c_1\sin(\alpha) + c_3\sinh(\alpha)$ and $0 = -c_1\sin(\alpha) + c_3\sinh(\alpha)$. Adding equations yields $2c_3\sinh(\alpha) = 0$ so $c_3 = 0$. (1 pt.) Substituting then leads to $0 = c_1\sin(\alpha)$. For nontrivial solutions to exist we must have $\sin(\alpha) = 0$ and consequently $\alpha = n\pi$ $(n = 1, 2, 3, \ldots)$ (1 pt.) Therefore $\boxed{\text{the negative eigenvalues of } T \text{ on } \mathbb{V} \text{ are } \lambda_n = -(n\pi)^4 \text{ and the corresponding eigenfunctions are } \varphi_n(x) = \cos(n\pi x) \text{ where } n = 1, 2, 3, \ldots}$ (1 pt.) (1 pt.)

2.(33 pts.) (a) Show that the Fourier cosine series of the function $f(x) = x^6 - 5x^4 + 7x^2$ on the interval $[0,1]$ is

$$\frac{31}{21} + \frac{1440}{\pi^6}\sum_{n=1}^{\infty}\frac{(-1)^n \cos(n\pi x)}{n^6}$$

(b) Write the partial sum $S_2 f(x)$ consisting of the first three terms in the Fourier cosine series of $f$. Sketch the graphs of $f$ and $S_2 f$ over the interval $[0,1]$ on the same set of coordinate axes. (To save time, raise your hand and I will come to your seat and grade your graphs from your calculator's display.)

(c) Discuss the convergence or lack thereof for the Fourier cosine series of $f$ on $[0,1]$. Be sure to give reasons for your answers for the three types of convergence: uniform, $L^2$, and pointwise.

(10) (a) The Fourier cosine series of $f$ on $(0,\ell)$ is $\sum_{n=0}^{\infty} a_n \cos\left(\frac{n\pi x}{\ell}\right)$ where

$a_0 = \frac{1}{\ell}\int_0^{\ell} f(x)\,dx$ and $a_n = \frac{2}{\ell}\int_0^{\ell} f(x)\cos\left(\frac{n\pi x}{\ell}\right)dx$ $(n=1,2,3,\ldots)$. With $f(x) = x^6 - 5x^4 + 7x^2$ and $\ell = 1$, we have coefficients

3 pts. to here. $a_0 = \int_0^1 (x^6 - 5x^4 + 7x^2)\,dx = \frac{x^7}{7} - x^5 + \frac{7}{3}x^3\Big|_0^1 = \frac{1}{7} - 1 + \frac{7}{3} = \frac{3-21+49}{21} = \frac{31}{21}$ and

5 to here $a_n = 2\int_0^1 (x^6 - 5x^4 + 7x^2)\cos(n\pi x)\,dx$ for $n \geq 1$. Write $\varphi^{(6)}(x) = \cos(n\pi x)$. Six integrations

9 pts. to here. by parts gives $\int_0^1 f(x)\varphi^{(6)}(x)\,dx = \left(f\varphi^{(5)} - f'\varphi^{(4)} + f''\varphi^{(3)} - f^{(3)}\varphi'' + f^{(4)}\varphi' - f^{(5)}\varphi\right)\Big|_0^1 + \int_0^1 f^{(6)}(x)\varphi(x)\,dx$.

Note that $\varphi^{(5)}(x) = \frac{\sin(n\pi x)}{n\pi}$ so $\varphi^{(5)}(1) = 0 = \varphi^{(5)}(0)$, $f'(x) = 6x^5 - 20x^3 + 14x$ so $f'(1) = 0 = f'(0)$,

$\varphi^{(3)}(x) = \frac{-\sin(n\pi x)}{(n\pi)^3}$ so $\varphi^{(3)}(1) = 0 = \varphi^{(3)}(0)$, $f^{(3)}(x) = 120x^3 - 120x$ so $f^{(3)}(1) = 0 = f^{(3)}(0)$, $\varphi'(x) = \frac{\sin(n\pi x)}{(n\pi)^5}$

so $\varphi'(1) = 0 = \varphi'(0)$, $f^{(5)}(x) = 720x$ so $f^{(5)}(0) = 0$, $\varphi(x) = \frac{-\cos(n\pi x)}{(n\pi)^6}$, and $f^{(6)}(x) = 720$. Therefore

13 pts. to here. $\left(f\varphi^{(5)} - f'\varphi^{(4)} + f''\varphi^{(3)} - f^{(3)}\varphi'' + f^{(4)}\varphi' - f^{(5)}\varphi\right)\Big|_0^1 = -f^{(5)}(1)\varphi(1) = \frac{720(-1)^n}{(n\pi)^6}$ and $\int_0^1 f^{(6)}(x)\varphi(x)\,dx$

14 pts. to here $= \frac{-720}{(n\pi)^6}\int_0^1 \cos(n\pi x)\,dx = 0$. Consequently,

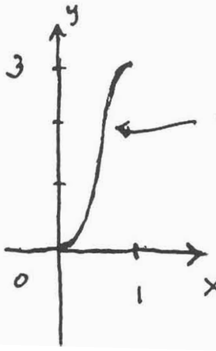
15 pts. to here. $$a_n = 2\int_0^1 f(x)\varphi^{(6)}(x)\,dx = \frac{1440(-1)^n}{(n\pi)^6} \quad \text{for } n \geq 1.$$

Thus

16 pts. to here. $$f(x) \sim \boxed{\frac{31}{21} + \frac{1440}{\pi^6}\sum_{n=1}^{\infty}\frac{(-1)^n\cos(n\pi x)}{n^6}}$$

is the Fourier cosine series representation for $f(x) = x^6 - 5x^4 + 7x^2$ on $[0,1]$.

(b) $S_2 f(x) = a_0 + a_1 \cos(\pi x) + a_2 \cos(2\pi x) = \boxed{\frac{31}{21} + \frac{1440}{\pi^6}\left( -\cos(\pi x) + \frac{\cos(2\pi x)}{64}\right)}.$




5 pts. to here.

$y = f(x) = x^6 - 5x^4 + 7x^2$

and $y = S_2 f(x)$ (The graphs are indistinguishable on $[0,1]$ to the resolution of an HP-49G calculator with window $0 \le x \le 1$ and $-1 \le y \le 4$.)

(12) (c) Note that $\Phi = \{\cos(n\pi x)\}_{n=0}^{\infty}$ is the complete orthogonal set of eigenfunctions for the eigenvalue problem $X''(x) + \lambda X(x) = 0$ with symmetric (Neumann) boundary conditions $X'(0) = 0$ and $X'(1) = 0$. Also $f(x) = x^6 - 5x^4 + 7x^2$, $f'(x) = 6x^5 - 20x^3 + 14x$, and $f''(x) = 30x^4 - 60x^2 + 14$ are continuous on $[0,1]$ and $f$ satisfies the symmetric boundary conditions that generate $\Phi$: $f'(0) = 0$ and $f'(1) = 0$. Then the uniform convergence result (Theorem 2) guarantees that the Fourier cosine series of $f$ converges uniformly to $f$ on $[0,1]$. Because uniform convergence (on bounded intervals, at least) implies $L^2$ convergence and pointwise convergence, the Fourier cosine series of $f$ converges to $f$ in the $L^2$ sense and the pointwise sense on $[0,1]$.

pts. to here. pts. to here. to here. pts. to here.

3.(34 pts.) Find a solution to

$$u_{tt} + u_{xxxx} \overset{①}{=} 0 \text{ for } 0 < x < 1,\ 0 < t < \infty,$$

subject to the boundary conditions

$$u_x(0,t) \overset{②}{=} 0 \overset{③}{=} u_x(1,t) \text{ and } u_{xxx}(0,t) \overset{④}{=} 0 \overset{⑤}{=} u_{xxx}(1,t) \text{ for } t \ge 0$$

and the initial conditions

$$u(x,0) \overset{⑦}{=} x^6 - 5x^4 + 7x^2 \text{ and } u_t(x,0) \overset{⑥}{=} 0 \text{ for } 0 \le x \le 1.$$

You may use any results stated in problems 1 and 2, even if you could not successfully solve those problems.

Bonus (10 pts.): Is there at most one solution to the above problem which is continuous on the strip $0 \le x \le 1$, $0 \le t < \infty$? Justify your answer.

We use the method of separation of variables. We seek nontrivial solutions to the homogeneous part of the above problem, ①-②-③-④-⑤-⑥, of the form $u(x,t) = X(x)T(t)$. Substituting in ① yields $X(x)T''(t) + X^{(4)}(x)T(t) = 0$ or $-\frac{X^{(4)}(x)}{X(x)} = \frac{T''(t)}{T(t)} = \text{constant} = \lambda$. Substituting in ②-③ gives $X'(0)T(t) = 0 = X'(1)T(t)$ for all $t \ge 0$. In order for the solution to be non-trivial we must have $X'(0) = 0 = X'(1)$. Similarly, substituting in ④-⑤ leads to $X^{(3)}(0) = 0 = X^{(3)}(1)$ and substituting in ⑥ yields $T'(0) = 0$. Therefore we are led to the system

2 pts.

$$X^{(4)}(x) + \lambda X(x) \overset{⑧}{=} 0,\quad X'(0) \overset{⑨}{=} 0 \overset{⑩}{=} X'(1) \text{ and } X^{(3)}(0) \overset{⑪}{=} 0 \overset{⑫}{=} X^{(3)}(1),$$

$$T''(t) - \lambda T(t) \overset{⑬}{=} 0,\quad T'(0) \overset{⑭}{=} 0.$$

(5 + 4 pts.)
(5 [illegible] pts.)
[illegible] pts. to here

18 By problem 1, the eigenvalues for ⑧-⑨-⑩-⑪-⑫ are $\lambda_n = -(n\pi)^4$ and the corresponding eigenfunctions are $X_n(x) = \cos(n\pi x)$ $(n = 0, 1, 2, \dots)$. Substituting in ⑬-⑭, we have

19 pts. to here

20

$$T_n''(t) + (n\pi)^4 T_n(t) \overset{⑬'}{=} 0,\quad T_n'(0) \overset{⑭'}{=} 0.$$

23 pts. to here. The general solution of ⑬′ is $T_n(t) = a_n\cos(n^2\pi^2 t) + b_n\sin(n^2\pi^2 t)$ when $n \ge 1$ and ⑭′ implies $b_n = 0$.

25 Therefore $u_n(x,t) = X_n(x)T_n(t) = \cos(n\pi x)\cos(n^2\pi^2 t)$ for $n = 1, 2, 3, \dots$ (up to a constant factor).

When $n = 0$, the general solution of ⑬′ is $T_0(t) = a_0 + b_0 t$ and ⑭′ implies $b_0 = 0$.

26 pts. to here. Therefore $u_0(x,t) = 1$ (up to a constant factor). The superposition principle gives

29

$$u(x,t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x)\cos(n^2\pi^2 t) \qquad (a_n\text{'s arbitrary constants for } n \ge 0)$$

as a formal solution to ①-②-③-④-⑤-⑥. We must choose the $a_n$'s so that ⑦ is satisfied:

$$x^6 - 5x^4 + 7x^2 = f(x) = u(x,0) = a_0 + \sum^{\infty} a_n \cos(n\pi x) \quad \text{for all } 0 \le x \le 1.$$

32 pts. to here. By problem 2 we should choose $a_0 = \frac{31}{21}$ and $a_n = \frac{1440(-1)^n}{(\pi n)^6}$ for $n \geq 1$. Thus

34 pts. to here.

$$\boxed{u(x,t) = \frac{31}{21} + \frac{1440}{\pi^6} \sum_{n=1}^{\infty} \frac{(-1)^n \cos(n\pi x)\cos(n^2\pi^2 t)}{n^6}}$$

solves (1)-(2)-(3)-(4)-(5)-(6)-(7).

<u>Bonus</u>: The solution above is the unique solution to (1)-(2)-(3)-(4)-(5)-(6)-(7). To see this, suppose that $u = v(x,t)$ were another solution to (1)-(2)-(3)-(4)-(5)-(6)-(7) that is

1 pt. to here. continuous for $0 \leq x \leq 1$ and $t \geq 0$. Consider $w(x,t) = u(x,t) - v(x,t)$. Then $w$ is continuous on $0 \leq x \leq 1$, $0 \leq t < \infty$, and solves the problem

2 pts. to here.

$$\begin{cases} w_{tt} + w_{xxxx} \overset{(15)}{=} 0 & \text{for } 0 < x < 1,\ 0 < t < \infty, \\ w_x(0,t) \overset{(16)}{=} 0 \overset{(17)}{=} w_x(1,t) \text{ and } w_{xxx}(0,t) \overset{(18)}{=} 0 \overset{(19)}{=} w_{xxx}(1,t) & \text{for } t \geq 0, \\ w(x,0) \overset{(20)}{=} 0 \overset{(21)}{=} w_t(x,0) & \text{for } 0 \leq x \leq 1. \end{cases}$$

4 pts. to here. Let $E(t) = \frac{1}{2}\int_0^1 \left[w_t^2(x,t) + w_{xx}^2(x,t)\right]dx$ be the total energy of the solution $w$ at $t \geq 0$.

Then $\frac{dE}{dt} \overset{(22)}{=} \int_0^1 \left[w_t(x,t)w_{tt}(x,t) + w_{xx}(x,t)w_{xxt}(x,t)\right]dx$. Integrating by parts twice gives

$$\int_0^1 w_{xx}(x,t)w_{xxt}(x,t)\,dx = \left(w_{xx}w_{xt} - w_{xxx}w_t\right)\Big|_{x=0}^{1} + \int_0^1 w_t(x,t)w_{xxxx}(x,t)\,dx.$$

Differentiating (16) and (17) with respect to $t$ produces $w_{xt}(0,t) \overset{(16')}{=} 0 \overset{(17')}{=} w_{xt}(1,t)$ for $t \geq 0$.

Therefore

$$\left(w_{xx}w_{xt} - w_{xxx}w_t\right)\Big|_{x=0}^{1} = w_{xx}(1,t)w_{xt}(1,t) - w_{xxx}(1,t)w_t(1,t) - w_{xx}(0,t)w_{xt}(0,t) + w_{xxx}(0,t)w_t(0,t)$$

$$= 0$$

by (17'), (19), (16'), and (18). Substituting in (22) leads to

$$\frac{dE}{dt} = \int_0^1 w_t(x,t)w_{tt}(x,t)\,dx + \int_0^1 w_t(x,t)w_{xxxx}(x,t)\,dx$$

6 pts. to here.

$$= \int_0^1 w_t(x,t)\left[w_{tt}(x,t) + w_{xxxx}(x,t)\right]dx = 0$$

by (15). Therefore $E(t) = E(0)$ for all $t \geq 0$. If we differentiate (20) twice with respect to $x$ we obtain $w_{xx}(x,0) \overset{(20')}{=} 0$ for all $0 \leq x \leq 1$. Then

8 pts. to here.

$$E(0) = \frac{1}{2}\int_0^1 \left[ w_t^2(x,0) + w_{xx}^2(x,0) \right] dx = 0$$

by (21) and (20'). Thus, for all $t \geq 0$,

$$0 = E(t) = \int_0^1 \left[ \tfrac{1}{2} w_t^2(x,t) + \tfrac{1}{2} w_{xx}^2(x,t) \right] dx$$

so the vanishing theorem implies $\frac{1}{2} w_t^2(x,t) + \frac{1}{2} w_{xx}^2(x,t) = 0$ for all $0 \leq x \leq 1$ and all $0 \leq t < \infty$, and hence $w_t(x,t) = 0 = w_{xx}(x,t)$. It follows that $w(x,t) = c_1 x + c_2$ for some constants $c_1$ and $c_2$ and all $0 \leq x \leq 1$, $0 \leq t < \infty$. But (20) implies $c_1 = 0 = c_2$. I.e. $u(x,t) - v(x,t) = 0$

10 to here.

for all $0 \leq x \leq 1$, $0 \leq t < \infty$, proving that $u = u(x,t)$ is the unique classical solution of (1)-(2)-(3)-(4)-(5)-(6)-(7).

# Convergence Theorems

Consider the eigenvalue problem

(1) $$X''(x)+\lambda X(x)=0 \text{ in } a<x<b$$

with any symmetric boundary conditions of the form

(2) $$\begin{cases}\alpha_1 f(a)+\beta_1 f(b)+\gamma_1 f'(a)+\delta_1 f'(b)=0\\ \alpha_2 f(a)+\beta_2 f(b)+\gamma_2 f'(a)+\delta_2 f'(b)=0\end{cases}$$

and let $\Phi=\{X_1,X_2,X_3,\ldots\}$ be the complete orthogonal set of eigenfunctions for (1)-(2). Let $f$ be any absolutely integrable function defined on $a\le x\le b$. Consider the Fourier series for $f$ with respect to $\Phi$:

$$\sum_{n=1}^{\infty} A_n X_n(x)$$

where

$$A_n=\frac{\langle f,X_n\rangle}{\langle X_n,X_n\rangle} \qquad (n=1,2,3,\ldots).$$

**Theorem 2.** (Uniform Convergence) If

(i) $f(x), f'(x)$, and $f''(x)$ exist and are continuous for $a\le x\le b$ and

(ii) $f$ satisfies the given symmetric boundary conditions,

then the Fourier series of $f$ converges uniformly to $f$ on $[a,b]$.

**Theorem 3.** ($L^2$ – Convergence) If

$$\int_a^b |f(x)|^2\,dx<\infty$$

then the Fourier series of $f$ converges to $f$ in the mean-square sense in $(a,b)$.

**Theorem 4.** (Pointwise Convergence of Classical Fourier Series)

(i) If $f$ is a continuous function on $a\le x\le b$ and $f'$ is piecewise continuous on $a\le x\le b$, then the classical Fourier series (full, sine, or cosine) at $x$ converges pointwise to $f(x)$ in the open interval $a<x<b$.

(ii) If $f$ is a piecewise continuous function on $a\le x\le b$ and $f'$ is piecewise continuous on $a\le x\le b$, then the classical Fourier series (full, sine, or cosine) converges pointwise at every point $x$ in $(-\infty,\infty)$. The sum of the Fourier series is

$$\sum_{n=1}^{\infty} A_n X_n(x)=\frac{f(x^+)+f(x^-)}{2}$$

for all $x$ in the open interval $(a,b)$.

**Theorem 4$\infty$.** If $f$ is a function of period $2l$ on the real line for which $f$ and $f'$ are piecewise continuous, then the classical full Fourier series converges to $\dfrac{f(x^+)+f(x^-)}{2}$ for every real $x$.

Math 325

Exam III

Summer 2011

$n = 24$

$\mu = 72.3$

$\sigma = 19.8$

| Range | Graduate Letter Grade | Undergraduate Letter Grade | Frequency |
|---|---|---|---|
| 87–100 | A | A | 7 |
| 73–86 | B | B | 6 |
| 60–72 | C | B | 4 |
| 50–59 | C | C | 4 |
| 0–49 | F | D | 3 |