Effect of Matrix Cracks on Damping in Unidirectional and Cross-Ply Ceramic Matrix Composites

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ABSTRACT: The paper elucidates the methods of estimating damping in ceramic matrix composites (CMC) with matrix cracks. Unidirectional composites with bridging matrix cracks and cross-ply laminates with tunneling cracks in transverse layers and bridging cracks in longitudinal layers are considered. It is shown that bridging matrix cracks dramatically increase damping in unidirectional CMC due to a dissipation of energy along damaged sections of the fiber–matrix interface (interfacial friction). Such friction is absent in the case of tunneling cracks in transverse layers of cross-ply laminates where the changes in damping due to a degradation of the stiffness remain small. However, damping in cross-ply laminates abruptly increases, if bridging cracks appear in the longitudinal layers.

KEY WORDS: ceramic matrix composites, matrix cracks, damping.

INTRODUCTION

THE PRESENT PAPER deals with the effect of matrix cracks in unidirectional and cross-ply CMC laminates on damping. As follows from experimental evidence [1–4], damping in ceramic matrix composites and ceramics is quite small. Damping increases in materials with damage and the problem that is addressed in this paper is related to a

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magnitude of this increase in unidirectional and cross-ply configurations. Note that the present analysis is conducted by assumption that new cracks are not formed during the motion (preexisting cracks).

It is useful to outline here a typical damage pattern in CMC. In the case of a unidirectional CMC material subject to axial tensile load, matrix cracks propagate in the planes perpendicular to the fibers [5]. These cracks may break the fiber, as they approach the fiber–matrix interface, but usually they continue their propagation in the matrix leaving behind the intact fiber and a section of the damaged fiber–matrix interface adjacent to the crack plane (''bridging cracks''). In the case of a cross-ply laminate, initial cracks appear in the transverse layers where they are parallel to the fibers (''tunneling cracks''). As the load continues to increase, these cracks reach saturation, and subsequently, bridging cracks appear in the longitudinal layers. Such sequence of cracking in cross-ply CMC laminates was observed in numerous experiments [6,7]. Note that the theoretical models for the damaged unidirectional and cross-ply CMC have been developed by numerous investigators (see, review paper [8] for details).

Some of the sources of an increased damping in laminated composites with matrix cracks are:

- Macromechanical stresses associated with the changes in stiffness due to cracking;
- Micromechanical stresses at the tips of the cracks;
- Energy dissipation due to thermomechanical coupling (see the review of this coupling phenomenon in [9]).

In addition, energy is dissipated due to friction along the damaged sections of the fiber–matrix interface adjacent to the matrix crack plane in unidirectional composites with bridging cracks that are perpendicular to the fibers. The same phenomenon is encountered in longitudinal layers of cross-ply laminates with bridging matrix cracks. Another source of energy dissipation is friction between the faces of a crack during a part of the motion cycle.

In the present paper, the emphasis is on the changes in damping associated with the macromechanical stress problem and with the interfacial friction. It is obvious that other sources will further increase the energy dissipation. Therefore, incorporating all involved sources can only reinforce the conclusion that damping increases due to the presence of matrix cracks. Accordingly, the results illustrated in the paper may serve as a lower limit for the loss factor and other damping characteristics.

The solution obtained in the paper is based on monitoring the variations in the loss factor of the material:

$$g = U_d / 2\pi U \tag{1}$$

where U_d is a density of energy dissipated per cycle of motion (energy per unit volume per cycle) and U is the maximum strain energy density during the cycle.

Note that if the loss factor is found, such damping characteristics as the specific damping capacity (ψ), the logarithmic decrement (δ), and the damping ratio (ζ) can be determined from [10]:

$$\psi = 2\delta = 4\pi\zeta = 2\pi g \tag{2}$$

The motion considered here represents reversed bending with the stress ratio $R = 0$. Accordingly, the applied composite stress varies as

$$\sigma_c = \sigma(1 + \sin \omega t) \tag{3}$$

where ω is the frequency of motion, and t is time. Compressive stresses are avoided in this analysis since their effect may be disruptive for the damping study due to a possible fiber microbuckling.

DAMPING IN UNIDIRECTIONAL CMC COMPONENTS WITH BRIDGING MATRIX CRACKS

Maximum Strain Energy Density

In a one-dimensional problem where the composite stress is acting along the fiber direction, the strain energy density is obtained from

$$U = \sigma_c^2/2E \tag{4}$$

where E is the modulus of elasticity in the fiber direction. This modulus was derived for the material with bridging cracks developed during lifetime by Pryce and Smith [11] as well as by a number of other investigators (see review of Birman and Byrd [8] for a comprehensive discussion). Byrd and Birman [12] modified the solution of Pryce and Smith to account for the case where bridging cracks were formed during processing. In the present work, we are concerned with the cracks formed during lifetime and employ the Pryce and Smith solution. The average per-cycle modulus of elasticity of a unidirectional brittle matrix material with bridging cracks undergoing cyclic loading is given in [13], based on the Pryce and Smith model (Figure 1):

$$E = \tau[\tau/E_L + (r/4s)(\Delta\sigma/E_f)(V_m E_m/V_f E_L)^2]^{-1} \tag{5}$$

In Equation (5), τ is an interfacial shear stress, r is the fiber radius, s is the preexisting matrix crack spacing, $\Delta\sigma$ is the range of tensile stresses applied to the layer, E_L is a

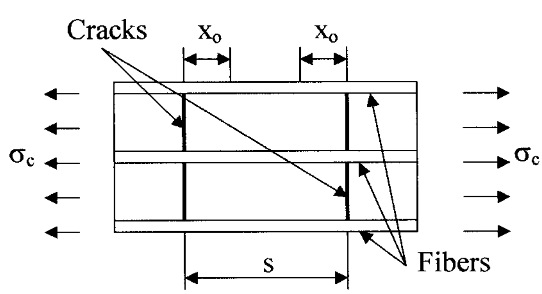

Figure 1. *Bridging cracks in a unidirectional CMC material. The matrix cracks spacing (s) and the length of the interfacial partial slip region (x_o) are shown in the figure.*

longitudinal modulus of the intact material, E_f and E_m are the moduli of fibers and matrix, respectively, and V_f and V_m are the volume fractions of the fibers and matrix, respectively.

Residual thermal stresses do not affect the average modulus E, as can easily be shown using the solution [11]. Note that the interfacial shear stress does not remain constant during fatigue cycling. Rather, it decreases with cycling due to wear, as was shown by Holmes and Cho [14]. Other factors, such as lubrication, strain rate, and temperature have also been shown to affect this stress. However, during small-amplitude steady state vibrations, the changes of the factors affecting the interfacial stress are slow. Accordingly, the value of the interfacial stress during one cycle may be assumed constant.

Equation (5) was obtained by assumption of a partial slip along the fiber–matrix interface during cycling. The limits of applicability of this equation are

$$E > V_f E_f/(1 - V_m E_m/2E_L) \tag{6}$$

If inequality (6) is violated, a partial-full slip occurs along the interface and Equation (5) should be replaced with a different relationship [13]. However, this situation is not considered here since a partial-full slip is unlikely in the case of small amplitude vibrations.

The maximum value of the strain energy density can be immediately obtained from Equation (4) using $\sigma_c = \Delta\sigma = 2\sigma$. Accordingly, in the case of a unidirectional material with bridging matrix cracks

$$U = (2\sigma^2/\tau)[\tau/E_L + (r/2s)(\sigma/E_f)(V_m E_m/V_f E_L)^2] \tag{7}$$

Damping in Intact Unidirectional CMC

The loss factor for a unidirectional composite without damage subject to axial loading was derived by Chang and Bert [15] in the form

$$g = (\lambda V_f g_f + V_m g_m)/(\lambda V_f + V_m) \tag{8}$$

where g_f and g_m are the loss factors for the fiber and matrix materials, respectively, and $\lambda = E_f/E_m$. Note that in the present solution the loss factors of the fibers and matrix are assumed independent of the stress amplitude. As is shown below, in the presence of bridging cracks, the energy dissipation associated with interfacial friction is dominant, i.e., any inaccuracies due to the above-mentioned assumption should be negligible.

Modifying a recently suggested formula [16], the loss factor can also be written as

$$g = (V_f g_f U_f + V_m g_m U_m)/(V_f U_f + V_m U_m) \tag{9}$$

where U_f and U_m are the strain energy densities of fibers (f) and matrix (m) at maximum vibratory displacements. Note that it is also possible to incorporate the energy of the interphase into Equation (9), although this contribution is neglected here for simplicity.

Using

$$U_f = \sigma_f^2/2E_f \quad U_m = \sigma_m^2/2E_m \tag{10}$$

and expressing the fiber and matrix stresses in terms of the maximum applied composite stress $\sigma_c = 2\sigma$ we obtain

$$\begin{aligned} U_f &= 2E_f\sigma^2/(V_fE_f + V_mE_m)^2 \\ U_m &= 2E_m\sigma^2/(V_fE_f + V_mE_m)^2 \end{aligned} \tag{11}$$

Now the loss factor given by Equation (9) yields the same result as Equation (8). The density of the energy dissipated per cycle of motion in the intact material is found as

$$U_d' = 2\pi g U \tag{12}$$

where g is obtained from Equation (8) and U is given by Equation (4) using the maximum value of stress, i.e., $\sigma_c = 2\sigma$, and the modulus of the intact material E_L.

Energy Dissipation in Unidirectional CMC with Bridging Matrix Cracks

The energy dissipation mechanism considered in this paper accounts for two contributions associated with matrix cracks, i.e., a modified amount of energy dissipation in the material with a reduced elasticity modulus, without accounting for interfacial friction, and the contribution associated with the friction along the damaged section of the fiber–matrix interface. These two contributions are assumed independent and considered separately.

The energy dissipation in a damaged material, without interfacial friction, can be estimated from Equation (8), accounting for a reduced effective modulus of elasticity of the matrix. In the case of vibrations of a damaged material, according to the rule of mixtures,

$$E = V_fE_f + V_mE_m' \tag{13}$$

where is E_m' an effective matrix modulus.

Equating the modulus given by Equation (13) to that in Equation (5) yields an effective matrix modulus:

$$E_m' = -V_fE_f/V_m + (\tau/V_m)[\tau/E_L + (r/4s)(\Delta\sigma/E_f)(V_mE_m/V_fE_L)^2]^{-1} \tag{14}$$

This modulus should be used in the coefficient λ in Equation (8), instead of the modulus E_m. Note that numerical examples presented below illustrate that the changes in damping due to the shear-lag effect and a reduced matrix stiffness are small and they may even be negligible, as compared to the changes associated with the interfacial friction.

The energy dissipation in a damaged material as a result of the interfacial friction can be estimated as follows. The frictional energy dissipation density per second due to the interfacial friction is obtained according to Cho et al. [17] as

$$w_{,t} = (fr/12s)[(\Delta\sigma)^3/E_f\tau](V_mE_m/V_fE_L)^2 \tag{15}$$

where f is the frequency of load (Hz). Accordingly, the energy dissipation density per cycle is obtained as

$$U_d'' = w_{,t}/f \tag{16}$$

This yields

$$U_d'' = (2r\sigma^3/3sE_f\tau)(V_mE_m/V_fE_L)^2 \tag{17}$$

Note that in a recent work of Birman and Byrd [18], a similar result was obtained based on the Pryce–Smith theory [11] and a simplified expression for a relative sliding between the fibers and matrix:

$$U_d'' = (5r\sigma^3/6sE_f\tau)(V_mE_m/V_fE_L)^2 \tag{18}$$

The difference between the results obtained by Equations (17) and (18) is 20%, i.e., both formulas yield qualitatively similar conclusions. Note that the dissipation energy (and the corresponding loss factor) associated with the interfacial friction is proportional to the third power of the stress amplitude.

The loss factor of the material with bridging matrix cracks can now be determined as

$$g_b = (U_d' + U_d'')/2\pi U = g' + g'' \tag{19}$$

where g' is given by Equation (8) using $E_m = E_m'$ and g'' is obtained from Equations (1), (7) and (17) or (18).

This accomplishes the theoretical part of the solution for a unidirectional CMC. It is interesting to note that the loss factor determined above is independent of the frequency of motion. However, it is clearly affected by the matrix crack spacing, interfacial shear stresses, applied stress amplitude, and the residual thermal stress. Accordingly, although there is not an explicit effect of frequency on damping in the solution presented above, the frequency affects damping via the amplitude of stresses that vary, dependent on the relationship between the driving and natural frequencies. Obviously, at resonance, the amplitude of motion and the stress in the structure reach maximum, increasing damping. At frequencies that are far from the natural frequencies of the structure the changes in the vibration amplitude as a function of frequency remain small and the effect of frequency on damping is weak. Notably, the conclusion of the negligible effect of vibration frequencies on damping in ceramics was obtained in the experimental study [19].

DAMPING IN CROSS-PLY CMC COMPONENTS WITH TUNNELING CRACKS IN TRANSVERSE LAYERS

Intact Cross-Ply CMC Material

The loss factor in 0-layers, i.e., g_0, can be evaluated from Equation (8). In transverse layers, the energy contributions in the fibers and in the matrix are given by

$$U_f = (1/2)V_f\sigma_f\varepsilon_f$$
$$U_m = (1/2)V_m\sigma_m\varepsilon_m \tag{20}$$

where both the stresses and the strains are acting in the transverse direction relative to the fibers.

According to the energy approach employed by Gibson [20], the transverse strains in the fibers and in the matrix can be found as

$$\varepsilon_f = a_2\varepsilon_c \quad \varepsilon_m = b_2\varepsilon_c \tag{21}$$

where the average transverse composite strain is ε_c. The values of coefficients in Equations (21) can be found from [20]:

$$a_2V_f + b_2V_m = 1$$
$$E_T = a_2^2E_{f2}V_f + b_2^2E_mV_m \tag{22}$$

In the above equations, E_{f2} is the fiber modulus of elasticity in the transverse direction and E_T is the transverse modulus of the composite material.

Substituting Equations (21) into (20) and using the formula for the loss factor of an arbitrary system of linear viscoelastic elements suggested by Ungar and Kerwin [21] that is reduced in the present case to Equation (9) one obtains

$$g_{90} = (g_fV_fa_2^2E_{f2} + g_mV_mb_2^2E_m)/(V_fa_2^2E_{f2} + V_mb_2^2E_m) \tag{23}$$

Note that Equation (23) does not employ the assumption that the transverse stresses in the fibers and matrix are equal to each other. As was shown in [20], this assumption that is used in the mechanics of materials approach to micromechanics of composite materials may lead to a noticeable inaccuracy in estimating transverse properties.

Now it is possible to evaluate the loss factor for the intact cross-ply material denoted here by g_{cp}. Using the approach similar to that in the case of a 0-layer, one obtains

$$g_{cp} = (g_0E_L + g_{90}E_T)/(E_L + E_T) \tag{24}$$

Stresses in Cross-ply CMC Laminates with Tunneling Matrix Cracks in Transverse Layers

As indicated above, the analysis is conducted by assumption that the cracks have already formed and new damage is not generated during motion. Therefore, the theory of Hahn and his associates [22] used here has to be modified as follows.

According to the approach employed in [22], a crack in the transverse layer is modeled through a superposition of the stresses in the laminate without a crack subject to a far-field stress σ_c and the stresses in the laminate with the crack subject to the stresses applied to the crack surfaces (Figure 2). The magnitude of the latter stresses is found based on the energy release consideration, i.e., it refers to the process of cracking, rather than the analysis of preexisting cracks. Contrary to the solution [22], in the present work, the cracks have already been formed, i.e., the stresses applied to the crack surfaces have to be determined from the analysis of the intact laminate and the requirement that the total stresses applied at the surface of the crack are equal to zero. Other assumptions employed in [22] are retained in the present analysis. Note that macromechanical thermal residual stresses considered in [22] are due to a mismatch between thermal expansion coefficients of longitudinal and transverse layers. As was shown in [23], these stresses are much smaller than their micromechanical counterparts (residual stresses due to a mismatch between the fiber and matrix materials), at least for the material considered in this paper. Accordingly, they are not considered here.

The solution for the intact laminate subject to an applied stress σ_c is trivial. The stresses in longitudinal and transverse layers of a balanced laminate (equal thickness of all layers) are given by

$$\sigma_0' = (E_L/E_i)\sigma_c \quad \sigma_{90}' = (E_T/E_i)\sigma_c \tag{25}$$

where $E_i = (E_L + E_T)/2$ is the modulus of the intact composite. Equation (25) is applicable to composite beams. In the case of a plate, the moduli in Equation (25) and in the subsequent transformations have to be replaced with the corresponding reduced stiffnesses, i.e., the Poisson effect should be taken into consideration. However, this correction does not alter the conclusions from the present study.

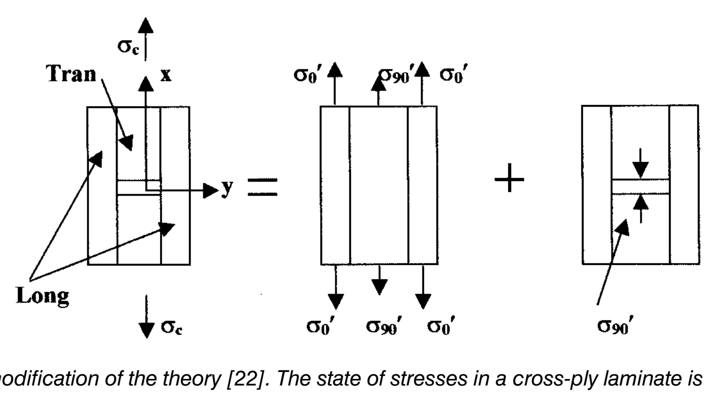

Figure 2. The modification of the theory [22]. The state of stresses in a cross-ply laminate is represented by a superposition of the solution for the intact laminate and the perturbed solution based on the requirement of stress-free crack surfaces. Notation: "Long" = longitudinal layers, "Tran" = transverse layer, σ_c = applied stress. Coordinate axes x and y are used in the subsequent discussion.

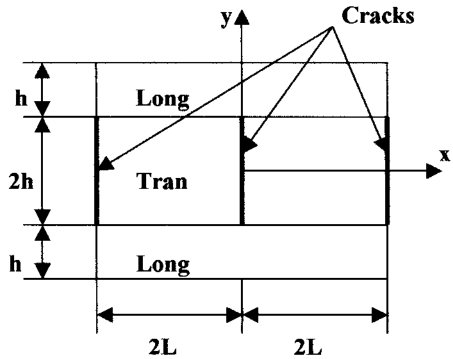

Figure 3. Geometry of a balanced cross-ply material with tunneling matrix cracks in transverse layers. Shown are a transverse layer with cracks and halves of two adjacent longitudinal layers. Notation: 2L = crack spacing, 2h = layer thickness.

Consider now the perturbed state of stresses due to the stress applied on the crack surfaces. The solution presented in [22] remains valid, except for the stress σ'_{90} that has already been obtained for the intact laminate in Equation (25). The solution of the governing differential equation for the perturbation stress in the 0-layers

$$\sigma_{0,xx} = (\phi/h)^2 \sigma_0 \tag{26}$$

has to be subject to the boundary conditions (Figure 3)

$$\begin{aligned} x = 0: &\quad u = 0,\ \sigma_0 = \sigma'_{90} \\ x = L: &\quad \sigma_{0,x} = 0,\ u_{,y} = 0 \end{aligned} \tag{27}$$

In Equations (26), (27), h is a layer half-thickness (note that it is assumed that all layers have equal thickness), L is a half-spacing of the tunneling cracks, and u is a displacement of the transverse layer in the x-direction defined in the theory [22] as a quadratic function of the y-coordinate:

$$u = f_1(x)y^2 + f_2(x) \tag{28}$$

where

$$\begin{aligned} f_{1,x} &= F_1\sigma_0 \quad f_{2,x} = F_2\sigma_0 \\ F_1 &= 3(E_L + E_T)/(2E_L E_T h^2) \\ F_2 &= 1/E_L - h^2 F_1 \end{aligned} \tag{29}$$

Furthermore, the shear-lag parameter is given by

$$\phi = [3G_T(E_L + E_T)/(E_L E_T)]^{1/2} \tag{30}$$

where G_T is the layer shear modulus in the plane perpendicular to the fibers.

The solution for the perturbation stresses is adopted from [22]:

$$\sigma_0 = A\sigma'_{90}[\exp(-\phi x_n) + \exp(-2\phi L_n)\exp(\phi x_n)]$$
$$\sigma_{90} = E_T A\sigma'_{90}[\exp(-\phi x_n) + \exp(-2\phi L_n)\exp(\phi x_n)](F_1 y^2 + F_2) \quad (31)$$

where

$$A = [1 + \exp(-2\phi L_n)]^{-1}$$
$$x_n = x/h \quad L_n = L/h \quad (32)$$

The total stress in the longitudinal and transverse layers can now be calculated as $(\sigma_0 + \sigma'_0)$ and $(\sigma_{90} + \sigma'_{90})$, respectively.

Loss Factor in Cross-Ply Laminates with and without Transverse Matrix Cracks

The loss factor is calculated below based on a comparison between the per-cycle dissipation energy and maximum strain energy. Both energy components are calculated for a representative half-cell, i.e., within $0 < x < L$ and $0 < y < 2h$ (see Figure 3). For the intact material, the maximum strain energy per unit width of the cross-ply beam is

$$U = U_0 + U_{90} = (h/2)\int_0^L [(\sigma'_0)^2/E_L + (\sigma'_{90})^2/E_T]dx \quad (33)$$

where U_0 and U_{90} denote the contributions of longitudinal and transverse layers, respectively, and the Poisson effect is neglected.

The same energy for the material with transverse cracks in the 90-layers is

$$U = U_0 + U_{90} = (h/2)\int_0^L [(\sigma_0 + \sigma'_0)^2/E_L dx + (1/2)\int_0^L \left[2\int_0^h (\sigma_{90}+\sigma'_{90})^2/E_T dy\right]dx \quad (34)$$

The composite loss factor can now be found from the formula that follows from the analysis in [10], i.e.,

$$g = (g_0 U_0 + g_{90} U_{90})/U \quad (35)$$

Note that in the case of the intact material, Equation (35) reduces to Equation (24).

In the presence of cracks, the substitution of Equations (25), (31) into the first integral in the right side of Equation (34), integration and transformations yield

$$U_0 = (h/2E_L)(\sigma_c/E_i)^2 L(E_L^2 + 2AE_L E_T \Phi_1 + A^2 E_T^2 \Phi_2) \quad (36)$$

where

$$\begin{aligned}\Phi_1 &= [1-\exp(-2\phi L_n)]/(\phi L_n)\\ \Phi_2 &= 2\,\exp(-2\phi L_n)+[1-\exp(-4\phi L_n)]/(2\phi L_n)\end{aligned} \tag{37}$$

Similar transformations yield

$$U_{90} = (h/2E_T)(\sigma_c E_T/E_i)^2 L(1+2K_1 A E_T \Phi_1 + K_2 A^2 E_T^2 \Phi_2) \tag{38}$$

where

$$\begin{aligned}K_1 &= F_1 h^2/3 + F_2\\ K_2 &= (F_1^2/5)h^4 + (2/3)F_1F_2h^2 + F_2^2\end{aligned} \tag{39}$$

The substitution of Equations (36), (38) into Equation (35) yields the following expression for the loss factor of a cross-ply material with tunneling matrix cracks in transverse layers:

$$g_T = (g_0 P_1 + g_{90} P_2)/(P_1+P_2) \tag{40}$$

where

$$\begin{aligned}P_1 &= 1+2A(E_T/E_L)\Phi_1 + A^2(E_T/E_L)^2\Phi_2\\ P_2 &= (1+2K_1AE_T\Phi_1 + K_2A^2E_T^2\Phi_2)E_T/E_L\end{aligned} \tag{41}$$

LOSS FACTOR IN CROSS-PLY CMC LAMINATES WITH TUNNELING MATRIX CRACKS IN TRANSVERSE LAYERS AND BRIDGING CRACKS IN LONGITUDINAL LAYERS

According to the numerical results presented below, the contribution of tunneling cracks in transverse layers to the change in damping can be disregarded. In this case, the loss factor in CMC laminates with cracks in all layers can be calculated as

$$g_{LT} = (0.5U_d'' + U_d''')/2\pi U \tag{42}$$

where the first term in the numerator that represents the density of energy dissipation in longitudinal layers due to interfacial friction has been defined by Equations (17) or (18) and U_d''' is the density of energy dissipation in the cross-ply material obtained neglecting friction. The factor 0.5 accounts for the fact that the volume of longitudinal layers is only half the volume of the component. The contribution to damping associated with the change in stiffness in the cross-ply material can be estimated from Equation (40) using the values of the moduli of elasticity of longitudinal and transverse layers ($E_L{}'$ and $E_T{}'$) that take into account the presence of matrix cracks in all layers. The technique for calculation of these moduli was outlined in [24].

As follows from the numerical results, an increase in the loss factor due to the change in stiffness is quite limited, even if matrix cracks appear in longitudinal layers. Therefore, it is possible to estimate the effect of matrix cracking using a simplified from of Equation (42):

$$g_{LT} = U''_d/4\pi U + g_{cp} \tag{43}$$

where g_{cp} is defined by Equation (24).

Note that although the dissipation energy density U''_d can be adopted from Equations (17) or (18) without changes, the stress that appears in these equations is the stress in the longitudinal layers. This stress is related to the applied composite stress σ_c by

$$\sigma_{\text{long}} = 2\sigma_c E'_L/(E'_L + E'_T) \tag{44}$$

The maximum strain energy density for the cross-ply material with matrix cracks subjected to the applied stress $\sigma_c = 2\sigma$ can be obtained as

$$U = 4\sigma^2/(E'_L + E'_T) \tag{45}$$

where the moduli are calculated at the stress $\sigma_c = 2\sigma$.

Now the first term in the right side of Equation (43) can be calculated. Note that it may be convenient to characterize the increase in damping in relative terms by introducing the ratio $R_g = g_{LT}/g_{cp}$. This ratio characterizes an increase in damping due to the presence of matrix cracks in all layers of a cross-ply CMC over damping in the intact material. The above-mentioned ratio can be reduced to the simple formula:

$$R_g = 1 + U''_d(E'_L + E'_T)/16\pi\sigma^2 g_{cp} \tag{46}$$

The expressions for the density of the interfacial energy dissipation is

$$U''_d = 2\pi g'' U_0 = 2\pi g''(\sigma_0^2/2E'_L) \tag{47}$$

where

$$\sigma_0 = \sigma_c(E'_L/E'_c) = 2\sigma(E'_L/E'_c) \tag{48}$$

where E'_c is the modulus of elasticity of the laminate with cracks in both transverse and longitudinal layers.

Substituting Equations (47) and (48) into Equation (46) yields

$$R_g = 1 + (g''/2g)(E'_L/E'_c) \tag{49}$$

NUMERICAL EXAMPLES AND DISCUSSION

Numerical Examples for Unidirectional CMC with Bridging Matrix Cracks

The representative material considered in the present study was SiC/CAS with the following properties [25]: $E_f = 200$ GPa, $E_m = 97$ GPa, $V_f = 0.35$, $r = 8 \times 10^{-6}$ m, the interfacial shear stress τ varied in the range between 5 and 17 MPa. Based on this data, the longitudinal modulus of the intact material is equal to $E_L = 133$ GPa. The saturation spacing of matrix cracks in this material is approximately 0.125 mm [25].

The loss factors for the material considered in the paper were estimated based on experimental data [1–4]. The loss factor for SiC fibers at room temperature reported in [1] is $g_f = 0.002$. The loss factor for CAS matrix could not be found in the open literature. However, based on data in [3] and [4], this factor is estimated as $g_m = 0.001$ (room temperature). Note that as is shown in the following examples, the knowledge of exact values of g_f and g_m is not very important for estimating the effect of bridging cracks on damping.

Small-amplitude free vibrations considered in this paper result in relatively low stresses in the composite material. These stresses in the examples were below the level that would result in additional cracks (the matrix cracking stress recorded in [25] was equal to 285 MPa). The additional limitation introduced in this paper is related to the assumption of a partial slip between the fibers and matrix (this assumption could be lifted, if necessary, using the solution for a damaged composite material corresponding to the partial-full slip). Computations carried out using the slip length according to the Pryce–Smith theory [11]

$$x_0 = (r/2\tau)(\sigma_c V_m E_m / V_f E_L) \tag{50}$$

and the partial-slip requirement $x_o < s/2$ yield the maximum stress amplitude during the cycle necessary to avoid full slip:

$$\sigma_c < 0.0923 \times 10^6 \tau s \tag{51}$$

If the interfacial shear stress $\tau = 17$ MPa and the crack spacing is taken equal to the saturation value $s = 0.125$ mm, Equation (51) yields the maximum stress per cycle equal to $\sigma_c = 2\sigma = 196$ MPa. With cycling, the interfacial shear stress decreases as a result of "smoothening" of the fiber–matrix interface. However, it remains quite high, so that the assumption of the partial slip is acceptable in most cases considered below.

The following results are shown for two components of the loss factor, i.e., g' obtained from Equation (8) and g'' obtained from Equations (1), (7) and (18). Obviously, the former loss factor represents a contribution associated with a decreased stiffness of the material with cracks, while the latter factor is related to the effect of interfacial friction. Using Equation (17), instead of Equation (18) would result in the loss factor g'' that is 20% smaller than the values shown in the following figures (this does not alter the conclusions). The results shown in Figure 4 for the loss factor g' illustrate that this factor increases as a result of both a higher range of applied composite stresses and due to a larger density of matrix cracks. Both results are predictable. A larger density of cracks (smaller spacing s) results in a more significant reduction of the matrix stiffness and accordingly, in a higher ratio $\lambda = E_f/E'_m$ and an increase of the loss factor obtained from Equation (8). Notably, a

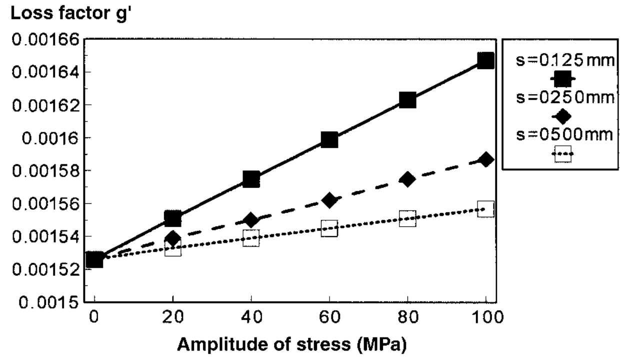

Figure 4. *Loss factor for a unidirectional CMC associated with the change in stiffness (g′) in the case* $\tau = 17\,MPa$.

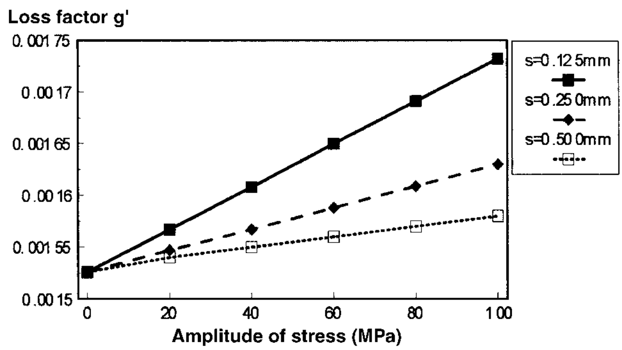

Figure 5. *Loss factor for a unidirectional CMC associated with the change in stiffness (g′) in the case* $\tau = 10\,MPa$.

similar dependence of damping on the crack density was reported by Plunkett [26]. An increase in damping as a result of a larger strain (and stress) range has been observed by Schultz and Warwick [27] and Gibson and Plunkett [28]. For CMC, experimental data reported in [11] results in a similar conclusion.

The interfacial shear stress is known to decline with the number of cycles [11]. A lower interfacial stress results in a higher damping [11]. These results are reflected in Figure 5 obtained for a lower interfacial shear stress than that in Figure 4, although the changes appear rather small. A much larger effect of the interfacial shear stress on damping is found below for the component of the loss factor directly associated with the interfacial shear.

The effects of the amplitude of the applied composite stress (σ) and the crack spacing on the loss factor g'' are shown in Figures 6, 7 and 8 for three different values of the interfacial shear stress. These values cover the range of the interfacial shear stress reported for the material considered in the paper. They vary from the highest value for the specimens where the cracks have just appeared to the lowest value corresponding to the specimens that have undergone fatigue loading. The conclusions from these figures coincide with some of the previously discussed results from Figures 4 and 5. In particular, a larger crack density (smaller spacing) results in an increase in damping. A decrease in the interfacial shear stress also yields a higher damping, implying an increase in energy

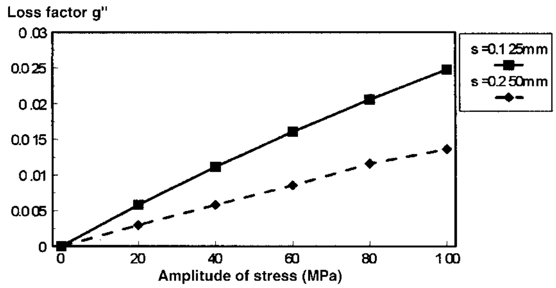

Figure 6. Loss factor for a unidirectional CMC associated with the interfacial friction (g″) in the case $\tau = 17$ MPa.

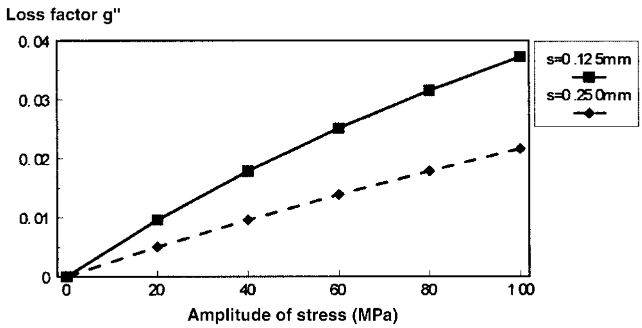

Figure 7. Loss factor for a unidirectional CMC associated with the interfacial friction (g″) in the case $\tau = 10$ MPa.

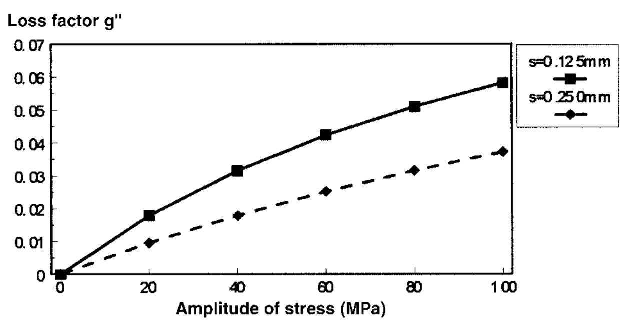

Figure 8. Loss factor for a unidirectional CMC associated with the interfacial friction (g″) in the case $\tau = 5$ MPa.

dissipation with fatigue cycling. The increase of damping with a higher stress level is in agreement with experimental data reported in [11].

Finally, the effect of the interfacial shear stress on the loss factor g'' is explicitly shown again in Figures 9 and 10 (note that the curve for the case where $s = 0.125$ mm and $\tau = 5$ MPa in Figure 10 is reliable only for the stress amplitudes less than 58 MPa, due to the full slip at higher stresses). The previous conclusion of a significant rise in damping with fatigue cycling (or with a decrease in the interfacial shear stress) is reflected in

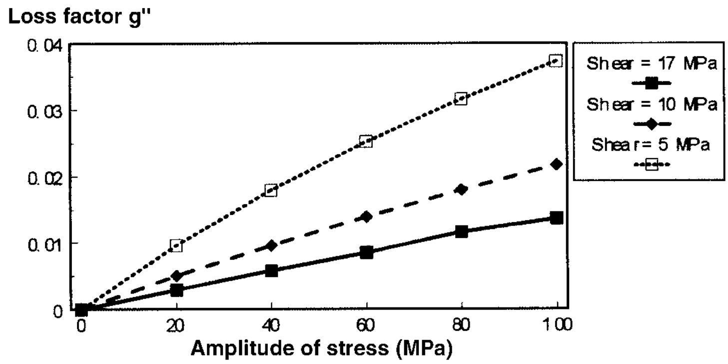

Figure 9. Effect of the interfacial shear stress on the loss factor for a unidirectional CMC associated with the interfacial friction (g″) in the case s = 0.250 mm.

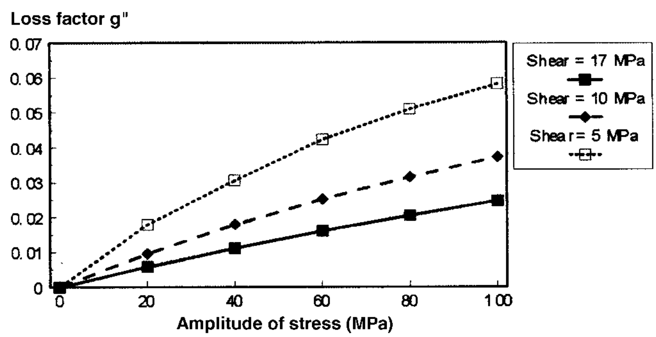

Figure 10. Effect of the interfacial shear stress on the loss factor on the loss factor for a unidirectional CMC associated with the interfacial friction (g″) in the case s = 0.125 mm.

these figures. In general, it is obvious from the results discussed above that the contribution to the loss factor associated with the interfacial friction is dominant if the stress amplitude is not too small (say, over 20 MPa). Therefore, the knowledge of exact values of the loss factors of the fibers and matrix that are necessary to estimate the contribution due to a reduced material stiffness is not critical since this factor is relatively small. This enables us to predict that in cross-ply CMC with tunneling matrix cracks the changes in damping due to the presence of damage will be negligible, unless the cracks penetrate into the longitudinal layers.

Numerical Examples for Cross-Ply CMC with Tunneling Matrix Cracks in Transverse Layers

The loss factor for a balanced cross-ply CMC laminate manufactured from the material described above was found equal to $g_{cp} = 0.001300$. The loss factor for the material with tunneling matrix cracks in transverse layers can be evaluated using the solution outlined above as a function of the crack spacing. The saturation spacing equal to 0.125 mm was reported in [25] for cross-ply laminates considered here with the layer thickness of

0.180 mm. Using the saturation spacing in the calculations of the loss factor illustrated that its deviation from the value corresponding to the intact cross-ply material was less than 1%. Considering inaccuracy in measurements of a relatively small damping in CMC, such minor variations can be disregarded, unless other sources of damping that are not considered in this paper indicate otherwise. The reason for such small damping change is obvious if we recall that even in unidirectional composites with bridging matrix cracks, an increase in damping due to a degradation in stiffness was very small compared to the counterpart attributed to the interfacial friction. In the case of tunneling cracks limited to transverse layers such friction is absent and a degradation in stiffness due to cracking is even smaller than a degradation in unidirectional components with bridging cracks.

Numerical Examples for Cross-Ply Laminates with Matrix Cracks in All Layers

As follow from Equation (49), the change in damping is affected by the ratio E_L'/E_c'. This ratio was evaluated as a function of the range of the applied composite stress in [24]. The analysis of this ratio illustrates that it remains nearly constant for a broad range of applied stress, if the matrix crack spacing in longitudinal layers is equal to or larger than 0.250 mm. Even if this spacing is quite small (0.125 mm), variations of the ratio E_L'/E_c' remain limited. For example this ratio decreases from 1.30 to 1.15 if the applied composite stress range changes from zero to 100 MPa and the transverse crack spacing is equal to $2L = 0.250$ mm. The corresponding decrease for the case where $2L = 0.500$ mm was from 1.19 to 1.11.

The following results are shown in Figure 11 for the ratios R_g and R_{gu} obtained as functions of the applied composite stress. The former ratio is introduced above and it reflects an increase in damping for a cross-ply material over damping in the intact laminate. The latter ratio is the corresponding increase in damping in unidirectional CMC with bridging matrix cracks of the same density. Several conclusions are available from this figure. As expected, the increase in damping in cross-ply configurations is smaller than

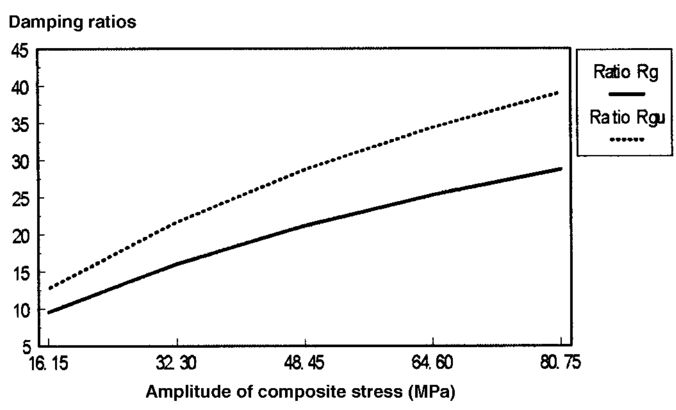

Figure 11. *The ratios of damping with and without matrix cracks in a cross-ply laminate (R_g) and in a unidirectional material (R_{gu}). The interfacial shear stress $\tau = 5$ MPa. Matrix crack spacing in longitudinal layers of the cross-ply laminate and in the unidirectional material is equal to 0.125 mm, matrix crack spacing in transverse layers is equal to 0.250 mm.*

in unidirectional materials. This is explained by the presence of transverse layers that experience a negligible change in damping. However, even in cross-ply laminates, the increase in damping due to matrix cracks is quite dramatic, if the cracks appear in longitudinal layers. The other interesting conclusion is that the ratio R_g/R_{gu} remains stable, particularly if the applied stress amplitude is large. Indeed, this ratio was approximately 0.74 for the entire range of stress amplitudes considered in Figure 11.

CONCLUSIONS

The paper outlines the analysis of damping in unidirectional and cross-ply ceramic matrix composites with matrix cracks. Numerical results presented in the paper result in the following conclusions.

Damping increases dramatically in unidirectional CMC with bridging matrix cracks, as compared to intact materials. The principal contribution to this increase is a dissipation of energy along the sections of the fiber–matrix interface damaged by bridging cracks. The contribution associated with the change in the stiffness due to cracking is much smaller and it may even be neglected compared to the interfacial energy dissipation. In this case the loss factor is proportional to the third power of the applied stress amplitude.

Damping changes in cross-ply CMC with tunneling matrix cracks in transverse layers are negligible. However, if the cracks appear in longitudinal layers, damping abruptly increases, mainly due to the interfacial friction. This increase is less dramatic than the changes in unidirectional materials. For example, in this paper the change in damping in a cross-ply material with matrix cracks in all layers was 74% of the change in unidirectional materials with the same spacing of bridging cracks.

Based on these results, it is possible to conclude that the changes in damping due to variations in the stiffness associated with matrix cracking are small and considering the fact that damping of ceramics and CMC is small, these changes are probably almost impossible to detect. However, damping increases dramatically, if the cracks damage the fiber–matrix interface. The associated increase in damping can easily be detected and attributed to the presence of such cracks.

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