AE/ME 339 Computational Fluid Dynamics (CFD)

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Grid Generation

Algebraic Methods

Known functions are used to map irregular physical domain into rectangular computational domains.

Example: Grid stretching may be necessary for some problems such as flow with boundary layers.
Let us consider the transformation:

$$\xi = x.....(5.50a)$$

$$\eta = \ln(y+1).....(5.50b)$$

Inverse transformation

$$x = \xi.....(5.51a)$$

$$y = \exp(\eta) - 1.....(5.51b)$$

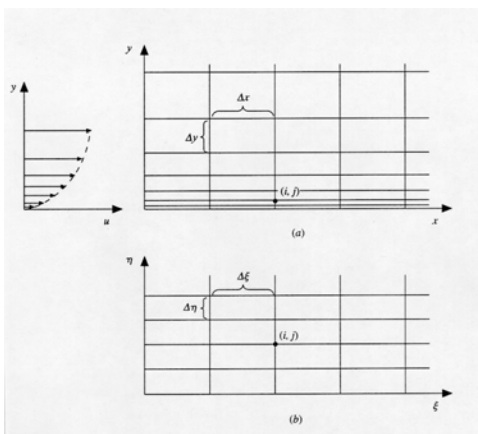

FIG. 5.4
Example of grid stretching. (a) Physical plane; (b) computational plane.

The following derivatives are used in the transformation

$$\frac{\partial \xi}{\partial x} = 1, \quad \frac{\partial \xi}{\partial y} = 0$$

$$\frac{\partial \eta}{\partial x} = 0, \quad \frac{\partial \eta}{\partial y} = \frac{1}{1+y}$$

$$x_\xi = 1, \quad y_\xi = 0$$

$$x_\eta = 0, \quad y_\eta = e^\eta$$

The following relation (Eq. 5.52) hold between increments Δy and Δη

$$\frac{dy}{d\eta} = e^\eta$$

$$dy = e^\eta d\eta$$

$$\Delta y = e^\eta \Delta \eta \qquad (5.52)$$

Therefore as η increases, Δy increases exponentially.
Thus we can choose Δη constant and still have an exponential stretching of the grid in the y-direction.

Transform the continuity equation as follows

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho \mathrm{v})}{\partial y}=0 ....(5.53)$$

$$\frac{\partial(\rho u)}{\partial \xi}\frac{\partial \xi}{\partial x}+\frac{\partial(\rho u)}{\partial \eta}\frac{\partial \eta}{\partial x}+\frac{\partial(\rho \mathrm{v})}{\partial \xi}\frac{\partial \xi}{\partial y}+\frac{\partial(\rho \mathrm{v})}{\partial \eta}\frac{\partial \eta}{\partial y}=0 ......(5.54)$$

Substitute for the derivatives in Eq. (5.54) to get

$$\frac{\partial\left(\rho u\right)}{\partial \xi}+\frac{1}{1+y}\frac{\partial\left(\rho \mathrm{v}\right)}{\partial \eta}=0 \qquad (5.56)$$

$$\frac{\partial\left(\rho u\right)}{\partial \xi}+\frac{1}{e^{\eta}}\frac{\partial\left(\rho \mathrm{v}\right)}{\partial \eta}=0 \qquad (5.57)$$

$$e^{\eta}\frac{\partial\left(\rho u\right)}{\partial \xi}+\frac{\partial\left(\rho \mathrm{v}\right)}{\partial \eta}=0$$

Eq. (5.57) is the continuity equation in the computational domain. Thus we have transformed the continuity equation from the physical space to the computational space.

Boundary Fitted Coordinate System (5.7)

Here we consider the flow through a divergent duct as given in Figure 5.6 (next slide). *de* is the curved upper wall and *fg* is the centerline.
Let $y_s = f(x)$ be the function that represents the upper wall.
The following transformation will give rise to a rectangular grid.

$$\xi = x \qquad (5.65)$$
$$\eta = y/y_{max} \qquad (5.66)$$

To test this choose $y_s = 1.5x$ and let x vary from 1 to 5.
At $x = 1$, $\xi = 1$, $\eta_{max} = 1$, and $x = 5$, $\xi = 5$, $\eta_{max} = 1$.

Thus the irregular domain is transformed into into a rectangular domain.

Case where the
Nozzle wall is curved

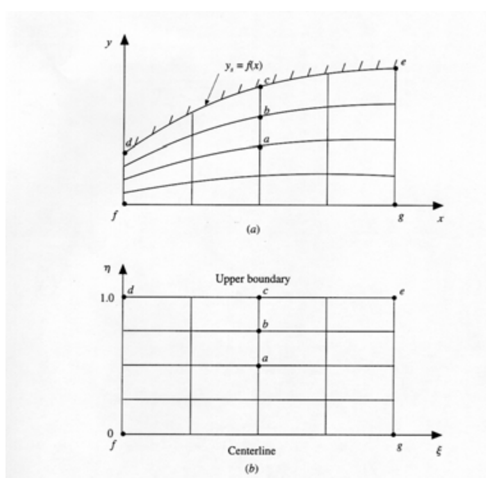

FIG. 5.6
A simple boundary-fitted coordinate system. (*a*) Physical plane; (*b*) computational plane.

Example:

$$y_{\max} = x^2 ....1 \le x \le 2$$

$$\xi = x$$

$$\eta = \frac{y}{y_{\max}} = \frac{y}{x^2}$$

$$\frac{\partial \xi}{\partial x} = 1, \qquad \frac{\partial \xi}{\partial y} = 0$$

$$\frac{\partial \eta}{\partial x} = -2\frac{y}{x^3} = -2\frac{\eta}{x} = -2\frac{\eta}{\xi}$$

$$\frac{\partial \eta}{\partial y} = \frac{1}{x^2} = \frac{1}{\xi^2}$$

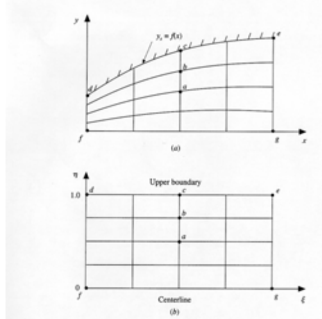

FIG. 5.6
A simple boundary-fitted coordinate system. (*a*) Physical plane; (*b*) computational plane.

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The above formulation is analytic

Could also be obtained using central differencing
if analytic expressions are not available

$$\frac{\partial(\xi,\eta)}{\partial(x,y)} \equiv \begin{vmatrix} \frac{\partial \xi}{\partial x} & \frac{\partial \xi}{\partial y} \\ \frac{\partial \eta}{\partial x} & \frac{\partial \eta}{\partial y} \end{vmatrix} \equiv \xi_x \eta_y - \xi_y \eta_x$$

Divide the domain into 4 segments each in the ξ and η directions
Therefore, $\Delta\xi = 0.25$ and $\Delta\eta = 0.25$
Consider a point in the domain where we have $\xi = 1.75$, $\eta = 0.75$
Let us calculate η_x analytically and numerically.

At this point:

$$x = 1.75$$

$$y = \eta y_{\max} = \eta x^2 = 0.75 \times (1.75)^2 = 2.29688$$

$$\eta = \frac{y}{x^2}$$

$$\eta_x = -2\frac{y}{x^3} = -2\frac{\eta}{\xi} = -2\times\frac{0.75}{1.75} = -0.85715$$

Numerical calculation using central differencing.
First calculate y at η = 0.5 and 1.0

$$y_\eta = \frac{\Delta y}{2\Delta\eta} = \frac{y_{\max}\eta_{j+1} - y_{\max}\eta_{j-1}}{2\Delta\eta} =$$

$$\frac{1.75^2\times 1.0 - 1.75^2\times 0.5}{2\times 0.25} = \frac{3.0625 - 1.53125}{2\times 0.25} = 3.0625$$

$$J = x_\xi y_\eta - y_\xi x_\eta = 1\times 3.0625 - 0\times 0 = 3.0625$$

$$\xi = 1.5 : y = y_{\max}\eta = 2.25\times 0.75 = 1.6875$$

$$\xi = 2 : y = y_{\max}\eta = 4\times 0.75 = 3.0$$

Numerically calculate the derivatives using CD

$$y_\xi = \frac{\Delta y}{\Delta \xi} = \frac{3.0 - 1.6875}{2.0 - 1.5} = 2.625$$

Using Eq. (5.35)

$$\eta_x = -\frac{y_\xi}{J} = -\frac{2.625}{3.0625} = -0.85714$$

$$\begin{bmatrix} \frac{\partial \xi}{\partial x} & \frac{\partial \xi}{\partial y} \\ \frac{\partial \eta}{\partial x} & \frac{\partial \eta}{\partial y} \end{bmatrix} = \frac{1}{J} \begin{bmatrix} \frac{\partial y}{\partial \eta} & -\frac{\partial x}{\partial \eta} \\ -\frac{\partial y}{\partial \xi} & \frac{\partial x}{\partial \xi} \end{bmatrix} \text{.....(5.35)}$$

Program Completed

University of Missouri-Rolla