AE/ME 339 Computational Fluid Dynamics (CFD)

K. M. Isaac

Example 2: Transform the quadrilateral domain (above) to a square domain (below) with its center at the origin

$$x = \left(\frac{1+\xi}{2}\right)\left(\frac{3-\eta}{2}\right)$$

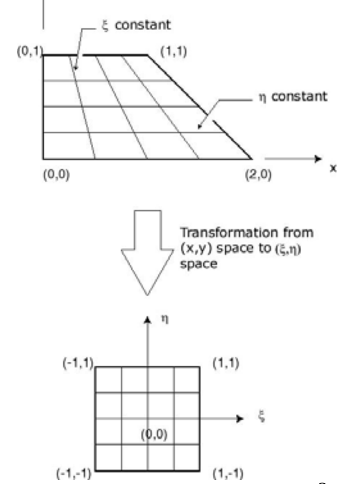
$$y = \left(\frac{\eta+1}{2}\right)$$

$$x_\xi = \frac{1}{4}(3-\eta)$$

$$x_\eta = -\frac{1}{4}(1+\xi)$$

$$y_\xi = 0$$

$$y_\eta = \frac{1}{2}$$



$$J = (x_\xi y_\eta - y_\xi x_\eta) = \frac{1}{4}(3-\eta)\frac{1}{2} - 0\times(...)$$

$$J = \frac{1}{8}(3-\eta)$$

See previous lectures for derivation of the following relations

$$\xi_x = \frac{y_\eta}{J} = \frac{1}{2}\frac{8}{(3-\eta)} = \frac{4}{(3-\eta)}$$

$$\xi_y = -\frac{x_\eta}{J} = -\frac{(1+\xi)}{4}\frac{8}{(3-\eta)} = -2\left(\frac{1+\xi}{3-\eta}\right)$$

$$\eta_x = -\frac{y_\xi}{J} = 0$$

$$\eta_y = \frac{x_\xi}{J} = \frac{(3-\eta)}{4}\frac{8}{(3-\eta)} = 2$$

Recall

$$x = \left(\frac{1+\xi}{2}\right)\left(\frac{3-\eta}{2}\right)$$

$$y = \left(\frac{\eta+1}{2}\right)$$

We can now use the above equations to see how the transformation looks like

$$x = 0, y = 0:$$

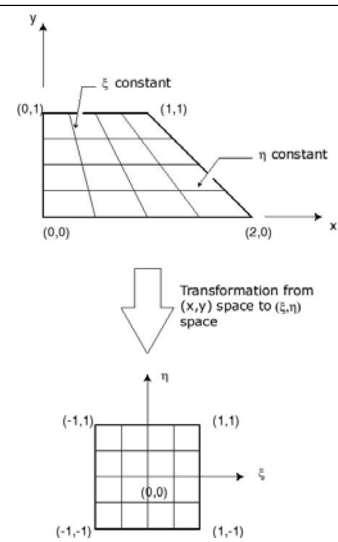
$$\frac{(1+\xi)}{2}\frac{(3-\eta)}{2} = 0$$

$$(1+\xi)(3-\eta) = 0$$

$$y = \left(\frac{\eta+1}{2}\right) = 0$$

$$\eta + 1 = 0 \Rightarrow \eta = -1$$

$$\therefore \xi = -1$$



$$x = 2, y = 0:$$

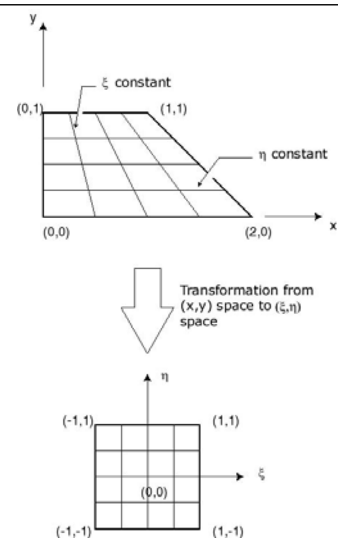
$$\frac{(1+\xi)}{2}\frac{(3-\eta)}{2} = 2$$

$$y = \left(\frac{\eta+1}{2}\right) = 0$$

$$(1+\xi)(3-\eta) = 8$$

$$\eta = -1$$

$$\therefore \xi = \frac{8}{4} - 1 = 1$$



Similarly:

at x = 0, y = 1: ξ = -1, η = 1

at x = 1, y = 1: x = 1, η = 1

Thus we get a rectangular computational domain centered at the origin.

Differential Equation Methods

If a partial differential equation is used to generate the grid, the properties of the solution can be used to control the grid properties.

All three types of PDEs have been used to generate CFD grids.

Elliptic Schemes

Elliptic PDEs have the property that the solutions are generally smooth.
Moreover, these equations govern potential flows.
To illustrate the properties of this method, consider potential flow over a cylinder.
The streamlines in this case are smooth and non-intersecting, and they, along with the potential lines, can be used as grid lines.
Grid spacing can be controlled by introducing an appropriate source term (solve Poisson’s equation instead of Laplace’s equation).

Figure: Streamlines and potential lines of flow over a cylinder

The desired grid points are chosen at the boundary of the physical domain and the differential equation is then solved to obtain the grid points in the interior of the domain.

$$\xi_{xx} + \xi_{yy} = P(\xi, \eta)$$
$$\eta_{xx} + \eta_{yy} = Q(\xi, \eta)$$

Program Completed

University of Missouri-Rolla