AE/ME 339 Computational Fluid Dynamics (CFD)

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$$x = \left(\frac{1+\xi}{2}\right)\left(\frac{3-\eta}{2}\right)$$

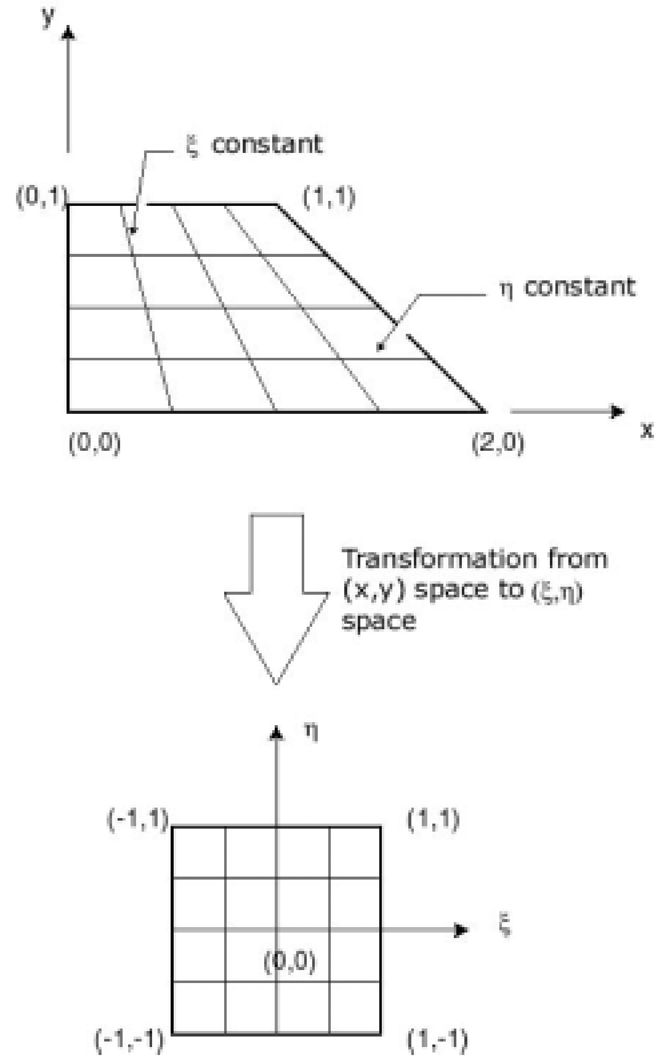
$$y = \left(\frac{\eta+1}{2}\right)$$

$$x_\xi = \frac{1}{4}(3-\eta)$$

$$x_\eta = -\frac{1}{4}(1+\xi)$$

$$y_\xi = 0$$

$$y_\eta = \frac{1}{2}$$



$$I = (J)^{-1} = (x_\xi y_\eta - y_\xi x_\eta) = \frac{1}{4}(3-\eta)\frac{1}{2} - 0 \times (\ldots)$$

$$I = \frac{1}{8}(3-\eta)$$

$$\xi_x = \frac{y_\eta}{I} = \frac{1}{2}\frac{8}{(3-\eta)} = \frac{4}{(3-\eta)}$$

$$\xi_y = -\frac{x_\eta}{I} = -\frac{(1+\xi)}{4}\frac{8}{(3-\eta)} = -2\left(\frac{1+\xi}{3-\eta}\right)$$

$$\eta_x = -\frac{y_\xi}{I} = 0$$

$$\eta_y = \frac{x_\xi}{I} = \frac{(3-\eta)}{4}\frac{8}{(3-\eta)} = 2$$

$$x = 0, y = 0:$$

$$\frac{(1+\xi)}{2}\frac{(3-\eta)}{2} = 0$$

$$(1+\xi)(3-\eta) = 0$$

$$\eta + 1 = 0 \Rightarrow \eta = -1$$

$$\therefore \xi = -1$$

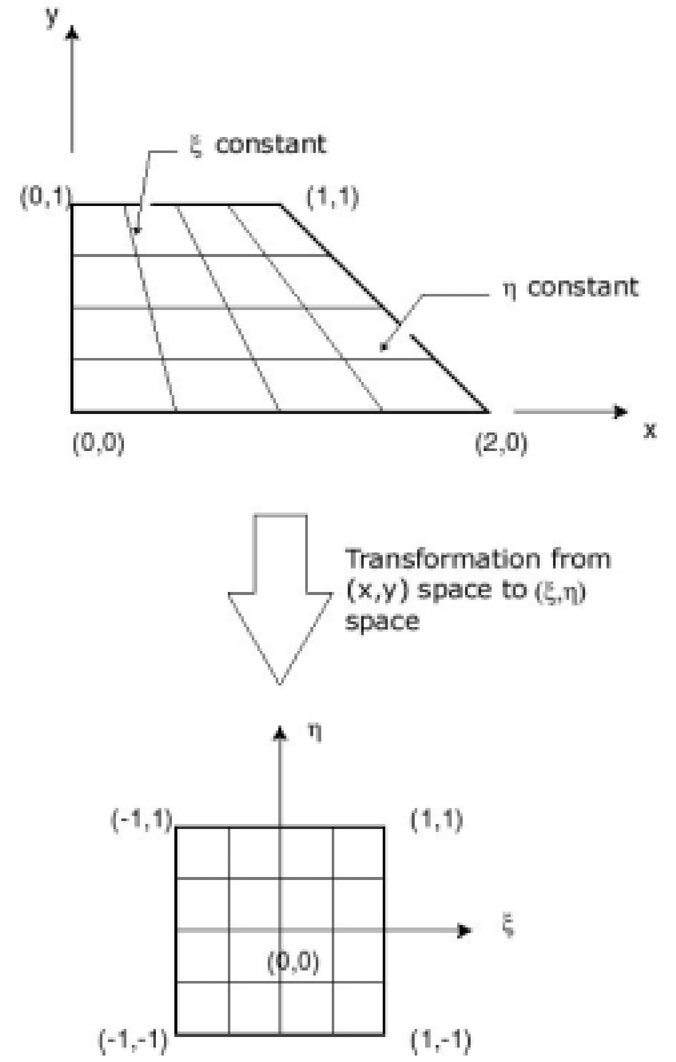
$$x = 2, y = 0:$$

$$\frac{(1+\xi)}{2}\frac{(3-\eta)}{2} = 2$$

$$(1+\xi)(3-\eta) = 8$$

$$\eta = -1$$

$$\therefore \xi = \frac{8}{4} - 1 = 1$$



Similarly:

at $x = 0$, $y = 1$: $\xi = -1$, $\eta = 1$

at $x = 1$, $y = 1$: $x = 1$, $\eta = 1$

Thus we get a rectangular computational domain centered at the origin.

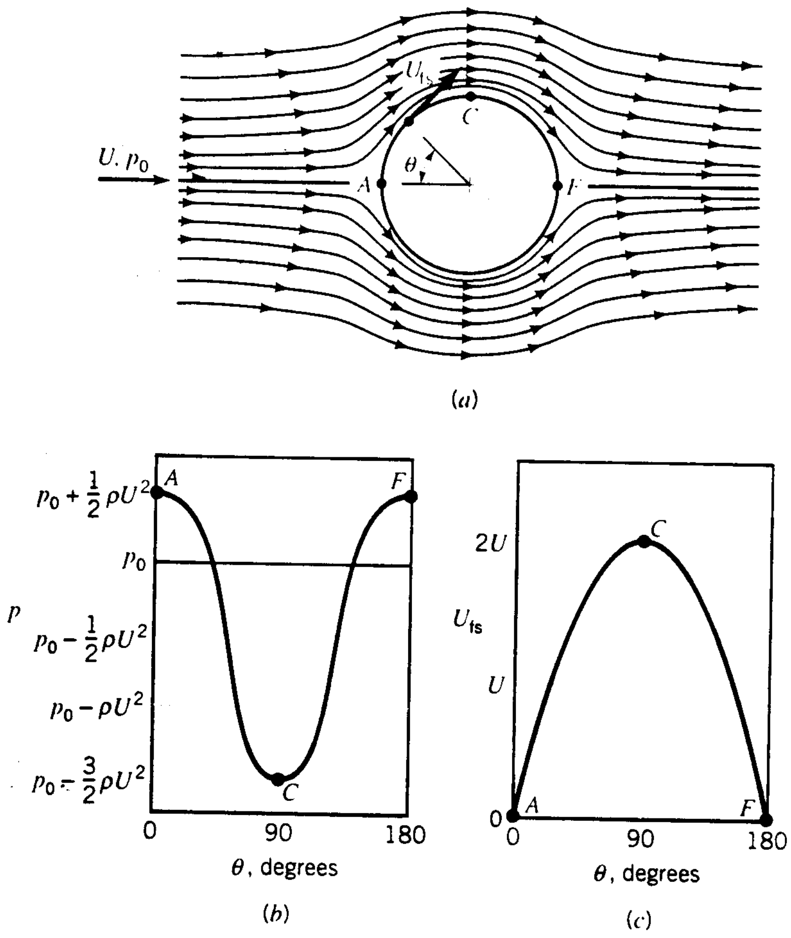
Differential Equation Methods

If a partial differential equation is used to generate the grid, the properties of the solution can be used to control the grid properties. All three types of PDEs have been used to generate CFD grids.

Elliptic Schemes

Elliptic PDEs have the property that the solutions are generally smooth.
Moreover, these equations govern potential flows.
To illustrate the properties of this method, consider potential flow over a cylinder.
The streamlines in this case are smooth and non-intersecting, and they along with the potential lines can used as grid lines.
Grid spacing can be controlled by introducing an appropriate source term (solve Poisson's equation instead of Lapace's equation).

Fig



■ FIGURE 9.16
Inviscid flow past a circu cylinder: (*a*) streamlines flow if there were no visc fects, (*b*) pressure distrib on the cylinder's surface, (*c*) free stream velocity o cylinder's surface.

The desired grid points are chosen at the boundary of the physical domain and the differential equation is then solved to obtain the grid points in the interior of the domain.

$$\xi_{xx} + \xi_{yy} = P(\xi, \eta)$$

$$\eta_{xx} + \eta_{yy} = Q(\xi, \eta)$$