AE/ME 339
Computational Fluid Dynamics (CFD)

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Basic parameter of the shock tube is the diaphragm pressure ration p*4*/p*1*.

The two chambers may be at different temperatures, T*1* and T*4*, and may contain differentgases having different gas constants, R*1* and R*4*.

At the instant when the diaphragm is broken, the pressure distribution is a step function. It then splits into a shock and an expansion fan as shown in the figure.

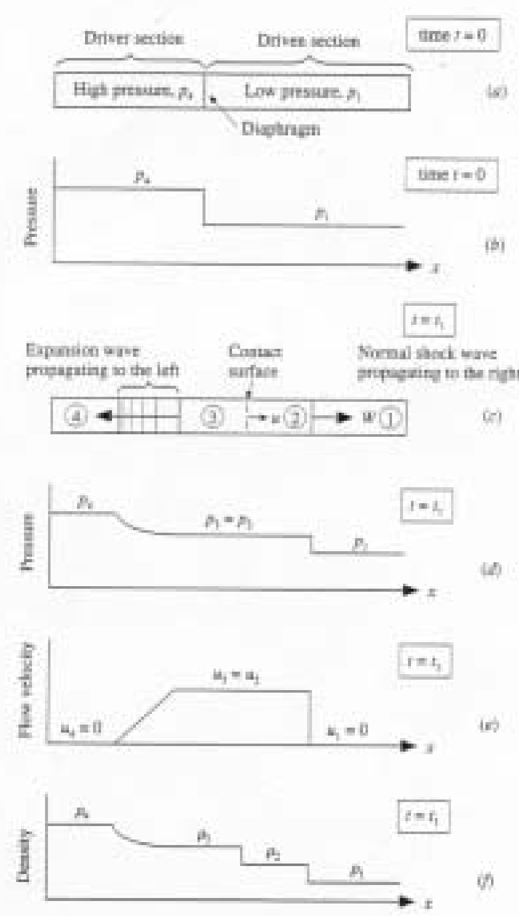

FIG. 11.5

The shock propagates into the expansion chamber with speed V_{shock}

An expansion wave propagates into the high pressure chamber with speed a_4 at its front.

Condition of the shock traversed by the shock is denoted by 2 and that traversed by the expansion wave is denoted by 3.

The interface between Regions 2 and 3 is called the *contact surface*. It marks the boundary between the fluids which were originally separated by the diaphragm. The contact surface is like the front of a piston driving into the low pressure region creating a shock front ahead of it.

The following conditions apply on either side of the contact surface

$$p_2 = p_3$$

and

$$u_2 = u_3$$

Temperatures and densities will be different in Regions 2 and 3.
The above two conditions are used to determine the shock strength p_3/p_4 and expansion strength p_2/p_1.

$$u_2 = a_1\left(\frac{p_2}{p_1} - 1\right)\sqrt{\frac{2/\gamma_1}{(\gamma_1+1)p_2/p_1 + (\gamma_1 - 1)}}$$

$$u_3 = \frac{2a_4}{\gamma_4 - 1}\left[1 - \left(\frac{p_3}{p_4}\right)^{(\gamma_4-1)/2\gamma_4}\right]$$

Equating the above two expression and using $p_3 = p_2$ gives

$$\frac{p_4}{p_1} = \frac{p_2}{p_1}\left[1 - \frac{(\gamma_4 - 1)(a_1/a_4)(p_2/p_1 - 1)}{\sqrt{2\gamma_1}\sqrt{2\gamma_1 + (\gamma_1+1)(p_2/p_1 - 1)}}\right]^{-2\gamma_4/\gamma_4 - 1}$$

The above expression gives shock strength p_2/p_1 implicitly as a function of the diaphragm pressure ratio p_4/p_1

The expansion strength can then be obtained as

$$\frac{p_3}{p_4} = \frac{p_3}{p_1}\frac{p_1}{p_4} = \frac{p_2 / p_1}{p_4 / p_1}$$

The temperature behind the shock is obtained from the Rankine-Hugoniot relations

$$\frac{T2}{T1} = \frac{1 + \dfrac{\gamma_1 - 1}{\gamma_1 + 1} \dfrac{p_2}{p_1}}{1 + \dfrac{\gamma_1 - 1}{\gamma_1 + 1} \dfrac{p_1}{p_2}}$$

$$\frac{\partial}{\partial t}\begin{Bmatrix} \rho \\ \rho u \\ \rho\left(e+\dfrac{u^2}{2}\right) \end{Bmatrix} + \frac{\partial}{\partial x}\begin{Bmatrix} \rho u \\ \rho u^2 + p \\ \rho\left(e+\dfrac{u^2}{2}\right)u + pu \end{Bmatrix} = 0 \ldots (A)$$