Computational Fluid Dynamics (AE/ME 339)
MAEEM Dept., UMR, Fall 2001

Home Work Problem 3 (modified RAM5) (**Retain 5 significant digits for your answers**)

Given that $f = (3x + 2^x).y$, do the following:

a) Using values of f from the above equation, calculate the backward, forward and central difference approximations of $\frac{\partial f}{\partial x}$ at x=1, y=1 for Δx values of 1.0, 0.5, 0.25, 0.1, 0.05, and 0.01. Report your results by constructing a table showing, for each value of Δx, the values of $f(x - \Delta x)$, $f(x + \Delta x)$ and the three finite difference approximations. Present your results as shown in the following table.

Δx	f(x-Δx) (exact)	f(x-Δx) (FD)	f(x+Δx) (exact)	f(x+Δx) (FD)
1				
0.5				
0.25				
0.1				
0.05				
0.01				

b) Using the equation for *f* to find the actual value of the first derivative, $\frac{\partial f}{\partial x}$ at (1,1), plot the error (defined as the difference between the exact value of the derivative and the finite difference approximation value) as a function of Δx for each type of difference. *Use a linear plot.*

c) Use the equation for *f* to determine the value of the first term truncated, as a function of Δx, for each of the three differences (FD, BD, CD). Use Taylor series to determine the value of the first term truncated.

d) Compare the values of these terms with the errors found in b), and discuss the extent to which the first term truncated provides an accurate representation of the truncation error.

Summary of procedure

1) Differentiate f and calculate exact values derivatives at (1,1).
2) Substitute in Taylor series to calculate f at (x+Delta_x) and f at (x-Delta_x) using the FD, BD and CD.
3) Plot error between the exact values and approximate values of f as a function of Delta_x.
4) Calculate the first truncated in Taylor series for each of the three methods. This will involve the exact value of the derivative.
5) Compare the results from (3) and (4). Discuss your conclusions.