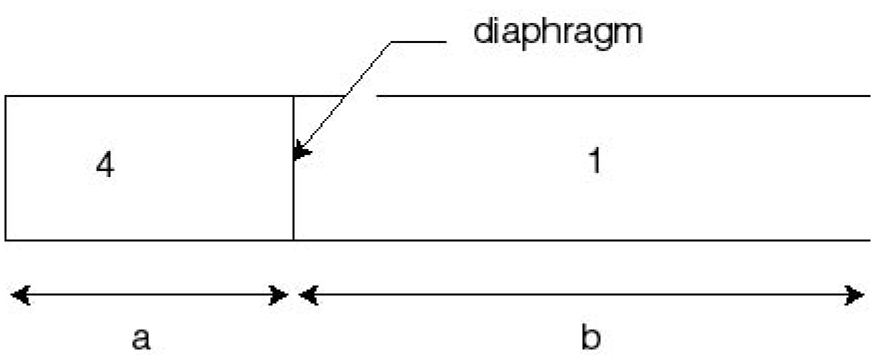
AE/ME 339

Homework Problem (RAM 18)
The Shock Tube

Using the MacCormack method, you are to determine the behavior of shock tube flow at various times and places. The shock tube is the one described in NACA TN 1903, configured as shown.



Shock Tube Schematic

The tube length is 200 cm. Results are to be obtained for two sets of equations: the 1-D inviscid equations, and the I-D inviscid equations with the artificial terms described in class. Assume that the fluid in the shock tube is air for which the specific heats are constant.

Initial conditions: $p_4 = 2$ atm, $p_1 = 1$ atm, $T_4 =$ 20C, $T_1 = 20C$

Three, dimensional Δx values are to be used: 2.5 cm, 1.25 cm, and 0.625 cm.
Use the CFL stability criterion

$$a\frac{\Delta t}{\Delta x} \leq 1$$

to choose the value of Δt. Note, *a* is the speed of sound.

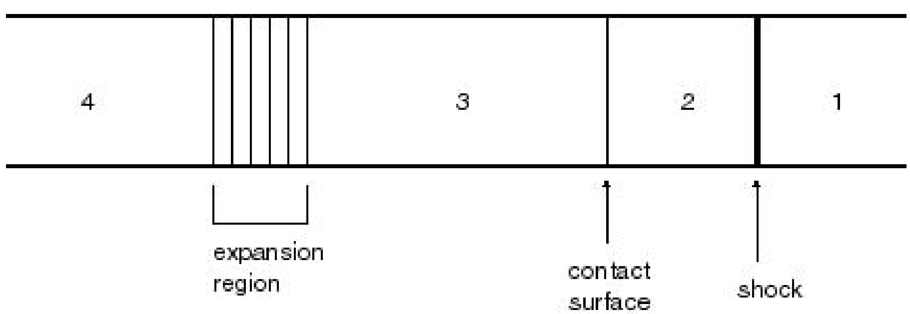
Dimensions in the first figure are: a = 80 cm, b = 120 cm.

You are to do the following:

1. Determine the spatial variation of p and u in the vicinity of the shock front at the times at which the shock has traveled 1/4, ½, and 3/4s of the way to the right end of the tube. <u>The closed form equation for the shock velocity is to be used to predict the times at which the shock is supposed to be at the locations given above.</u> This equation, and the other related equations, will provide you with some values which you can use to check the accuracy of your results.

2. Determine the variation of p and u throughout the entire length of the shock tube at 1 millisecond. Your discussion is to include a comparison between your results and the analytical results.

3. Provide a general write-up of your results which, in addition to the items mentioned above, is to include: a program listing; a discussion of the differences in the results obtained from the two sets of equations; a discussion of the effects of reducing Δx. Program output is to include all parameter values, the initial values, and the values of all quantities, suitably labeled, necessary to cover the items listed above. Plots can, and should, be used as appropriate. ORGANIZE YOUR REPORT SO AS KEEP THE NUMBER OF PAGES TO A MINIMUM.



Shock Tube Process

Shock Tube Analytical Results and other Useful Formulas

$$V_{shock} = a_1 \left[\frac{\gamma - 1}{2\gamma} + \left(\frac{\gamma + 1}{2\gamma} \right) \frac{p_2}{p_1} \right]^{1/2}$$

$$\frac{p_4}{p_1} = \frac{p_2}{p_1}\left[1 - \frac{(\gamma - 1)\left(\frac{a_1}{a_4}\right)\left(\frac{p_2}{p_1} - 1\right)}{\sqrt{2\gamma}\sqrt{2\gamma + (\gamma + 1)\left(\frac{p_2}{p_1} - 1\right)}}\right]^{-\left(\frac{2\gamma}{\gamma - 1}\right)}$$

$$u_2 = u_3 = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left[\frac{\frac{2\gamma}{\gamma + 1}}{\frac{p_2}{p_1} + \frac{\gamma - 1}{\gamma + 1}}\right]^{1/2}$$

$$a = \sqrt{\gamma R T}$$

$$\gamma = \frac{c_p}{c_v} = 1.4$$

$$p = \rho R T$$