AE/ME 339 Computational Fluid Dynamics (CFD)

K. M. Isaac

Basic parameter of the shock tube is the diaphragm pressure ration p4/p1.

The two chambers may be at different temperatures, T1 and T4, and may contain differentgases having different gas constants, R1 and R4.

At the instant when the diaphragm is broken, the pressure distribution is a step function. It then splits into a shock and an expansion fan as shown in the figure.

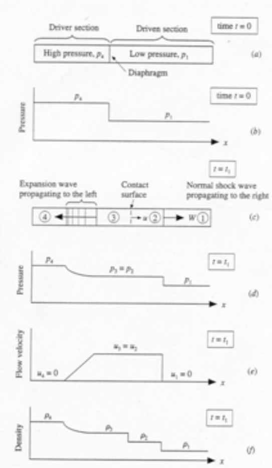

FIG. 11.5

The shock propagates into the expansion chamber with speed Vshock
An expansion wave propagates into the high pressure chamber with speed a4 at its front.
Condition of the shock traversed by the shock is denoted by 2 and that traversed by the expansion wave is denoted by 3.

The interface between Regions 2 and 3 is called the *contact surface*. It marks the boundary between the fluids which were originally separated by the diaphragm. The contact surface is like the front of a piston driving into the low pressure region creating a shock front ahead of it.

The following conditions apply on either side of the contact surface

$$p_2 = p_3$$

and

$$u_2 = u_3$$

Temperatures and densities will be different in Regions 2 and 3.
The above two conditions are used to determine the shock strength p_3/p_4 and expansion strength p_2/p_1.

$$u_2 = a_1\left(\frac{p_2}{p_1} - 1\right)\sqrt{\frac{2/\gamma_1}{(\gamma_1+1)\,p_2/p_1 + (\gamma_1 - 1)}}$$

$$u_3 = \frac{2a_4}{\gamma_4 - 1}\left[1 - \left(\frac{p_3}{p_4}\right)^{(\gamma_4-1)/2\gamma_4}\right]$$

Equating the above two expression and using $p_3 = p_2$ gives

$$\frac{p_4}{p_1} = \frac{p_2}{p_1}\left[1 - \frac{(\gamma_4 - 1)(a_1/a_4)(p_2/p_1 - 1)}{\sqrt{2\gamma_1}\sqrt{2\gamma_1 + (\gamma_1+1)(p_2/p_1 - 1)}}\right]^{-2\gamma_4/\gamma_4 - 1}$$

The above expression gives shock strength p2/p1 implicitly as a function of the diaphragm pressure ratio p4/p1
The expansion strength can then be obtained as

$$\frac{p_3}{p_4} = \frac{p_3}{p_1}\frac{p_1}{p_4} = \frac{p_2/p_1}{p_4/p_1}$$

The temperature behind the shock is obtained from the Rankine-Hugoniot relations

$$\frac{T_2}{T_1} = \frac{1 + \dfrac{\gamma_1 - 1}{\gamma_1 + 1}\dfrac{p_2}{p_1}}{1 + \dfrac{\gamma_1 - 1}{\gamma_1 + 1}\dfrac{p_1}{p_2}}$$

$$\frac{\partial}{\partial t}\begin{Bmatrix} \rho \\ \rho u \\ \rho\left(e + \dfrac{u^2}{2}\right) \end{Bmatrix} + \frac{\partial}{\partial x}\begin{Bmatrix} \rho u \\ \rho u^2 + p \\ \rho\left(e + \dfrac{u^2}{2}\right)u + pu \end{Bmatrix} = 0 ...(A)$$