ME/AE 339

Computational Fluid Dynamics

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Topic1b_ODE

R-K Method

Equation governing 2D jet flow

$$\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} + \left(\frac{df}{d\eta}\right)^2 = 0$$

Numerical solution can be obtained using Runge-Kutta method.
Outline:

1. Rewrite the above as three first order equations
2. Cast as an initial-value problem.
3. Solve using Runge-Kutta

Define

$$\frac{df}{d\eta} = \xi, \quad \frac{d\xi}{d\eta} = \zeta$$

The 3 first-order equations become

$$\frac{df}{d\eta} = \xi$$

$$\frac{d\xi}{d\eta} = \zeta$$

$$\frac{d\zeta}{d\eta} = -f\zeta + \xi^2$$

Boundary Conditions:

at $\eta = 0$:

$f = 0, \quad \zeta = 0$

at $\eta = \infty$:

$\xi = 0$

The above is a boundary-value problem. In order to use a method such as R-K, the problem must be recast as an I-V problem. Instead of applying the third BC directly, solution is obtained for different values of ξ at $\eta = 0$.
The correct solution is the one that satisfies the third boundary condition.

Example

Consider the RLC circuit which is governed by the following equation

$$LC\frac{d^2V}{dt^2} + RC\frac{dV}{dt} + V = 0$$

where

L - inductance (Henry)

C - capacitance (Farad)

R - resistance (Ohm)

V - voltage (volt)

Initial conditions:

$$V(0) = V_0, \quad \frac{dV}{dt}(0) = 0$$

Rewrite the circuit equation as

$$LC\frac{d^2V}{dt^2} = -RC\frac{dV}{dt} - V$$

The above second-order equation can be written as the two following first-order equations:

$$Y_1 = V, \quad Y_2 = \frac{dV}{dt}$$

$$\frac{dY_1}{dt} = \frac{dV}{dt} = Y_2, \quad \frac{dY_2}{dt} = \frac{d^2V}{dt^2} = -\frac{R}{L}\frac{dV}{dt} - \frac{1}{LC}V = -\frac{R}{L}Y_2 - \frac{1}{LC}Y_1$$

with following initial conditions:

$$Y_{1,0} = V_0, \quad Y_{2,0} = 0$$