Differential Equations and Dynamical Systems
https://doi.org/10.1007/s12591-019-00492-w

ORIGINAL RESEARCH

Control of Wheeled Mobile Robots on Time Scales

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Abstract
This paper deals with the control of unmanned mobile robots which are modeled by three-dimensional systems of first order dynamic equations. Our goal is to show the asymptotic stability of the zero solution of the system. It turns out that the results in the continuous case can be improved by proposing different controllers. Therefore, we are able to unify the results and extend them to one comprehensive theory, called time scale theory which can be accepted beyond the continuous and discrete cases.

Keywords Stability · Stability on time scales · Lyapunov · Invariance principles

Mathematics Subject Classification 93D05 · 93C10 · 93C95 · 34N05

Introduction

In recent years, intelligent controller techniques such as neural network controllers are widely used. One of the main drawback of these controllers is the computational complexity. Therefore, event-triggered controllers [3,6,7,10,11] become popular wherein execution time of the controllers is based on the real-time operation of the system. The main purpose of the event-triggered controller is to reduce the computation cost of the controller which can be highly benefitial for unmanned vehicle control applications since they only have limited built-in microprocessors to execute the controller. In the event-triggered controller, an event-triggering condition is designed by taking into account the stability and the closed-loop performance of the systems. However, the controller has to check if the event condition is

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Published online: 20 September 2019

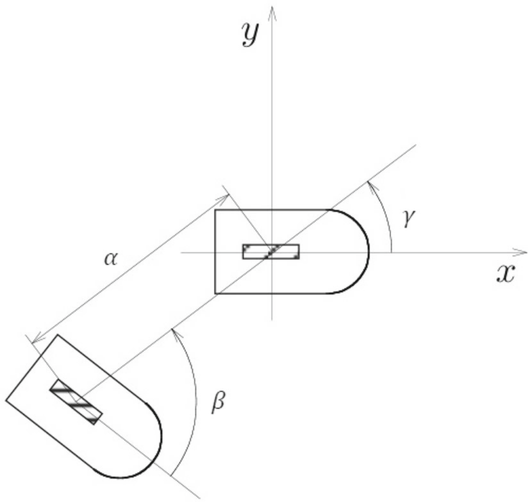

Fig. 1 Definition of the States for the Unicycle

satisfied or not continuously which also causes computational cost. Therefore, a novel regulation controller is defined for a nonholonomic mobile robot in a generalized time scale. This new approach enables the user to calculate the controller at some instants similar to the event triggered approaches [3,6,7,10,11]. The benefit of the controller defined in a general time scale over the event triggered controllers is that the event triggering condition is not necessarily be checked which reduces the computation further. Most recently, there are new advances in control of ground vehicles and mobile robots, see [8,9]. For example, Sun, Zhang and Liu [9] provides a two-time scale ABS control scheme for ground vehicles without knowing the priori knowledge of the road condition. Also, Sun, Tang, Gao and Zhao [9] propose a controller for mobile robots to improve the transient performance by using time-scale filtering technique.

A unicycle is a vehicle with a single orientable wheel, which is also the model of a nonholonomic Wheeled Mobile Robot (WMR) and corresponds to a single wheel rolling on the plane. Consider the system

$$\begin{cases} \alpha^{\Delta}(t) = -v(t)\cos\beta(t) \\ \beta^{\Delta}(t) = \frac{\sin\beta^{\sigma}(t)}{\alpha^{\sigma}(t)}v(t) - w(t) \\ \gamma^{\Delta}(t) = \frac{\sin\beta^{\sigma}(t)}{\alpha^{\sigma}(t)}v(t), \end{cases} \tag{1}$$

on a time scale $\mathbb{T}$, a nonempty closed subset of real numbers, where α is the distance of the reference point (x, y) of the unicycle from the goal (origin), β is the angle of the pointing vector to the goal with respect to the unicycle main axis, γ is the angle of the same pointing vector with respect to x axis, v and w are the controllers, see Fig. 1 ([5]).

System (1) in the case $\mathbb{T} = \mathbb{R}$ is considered in [5] by Luca, Oriola and Vendittelli to show the asymptotic stability of the zero solution. However, the control inputs proposed in [5] do not allow us to unify the results on general time scales. The proposed controllers given by [5] enables us to show the asymptotic stability in continuous case by using the Barbalat's Lemma. However, since the Barbalat's lemma does not exist in general time scale theory, we provide a new controller to show the asymptotic stability without any need of Barbalat's lemma. Therefore, the results are unified and extended into one comprehensive theory.

The structure of this paper is as follows: In Sect. 2, we give the preliminary results of the time scale calculus and stability theory for the interested readers to understand the basis of the time scale theory. Section 3 provides the asymptotic stability of the zero solution of system (1) with different controllers. In Sect. 4, we demonstrate the simulation results in order to validate our theoretical claims. Finally, we give a conclusion in the last section.

Preliminary Results

In this section, we give the preliminary results on time scales and basic definitions and theorems on stability and invariance principle.

Time Scales Calculus

The theory of time scales is initiated by Stefan Hilger in his PhD thesis in 1988 in order to unify discrete and continous analysis and combine them in one comprehensive theory.

Definition 1 [1, Definition 1.1] For $t \in \mathbb{T}$, $\sigma : \mathbb{T} \to \mathbb{T}$ is called the *forward jump operator* defined by

$$\sigma(t) := \inf\{s \in \mathbb{T} : \quad s > t\}$$

while $\rho : \mathbb{T} \to \mathbb{T}$ is called the *backward jump operator* given by

$$\rho(t) := \sup\{s \in \mathbb{T} : \quad s < t\}.$$

Finally, $\mu : \mathbb{T} \to [0, \infty)$ is called the *graininess function* given by $\mu(t) := \sigma(t) - t$.

The classification of points on time scales are very significant. For example, t is said to be *left-scattered* when $\rho(t) < t$, while *right-scattered* when $\sigma(t) > t$. We say that t is *isolated* when t is left and right scattered at the same time. In addition, we call t *right-dense* when $t < \sup \mathbb{T}$ and $\sigma(t) = t$, while t is called *left-dense* when $t > \inf \mathbb{T}$ and $\rho(t) = t$. Finally, we call t *dense* when t is right and left-dense at the same time. If $\sup \mathbb{T} < \infty$, then $\mathbb{T}^\kappa = \mathbb{T} \backslash (\rho(\sup \mathbb{T}), \sup \mathbb{T}]$, and $\mathbb{T}^\kappa = \mathbb{T}$ if $\sup \mathbb{T} = \infty$. Also $f^\sigma : \mathbb{T} \to \mathbb{R}$ is defined by $f^\sigma(t) = f(\sigma(t))$ for all $t \in \mathbb{T}$, where $f : \mathbb{T} \to \mathbb{R}$.

Definition 2 [1, Definition 1.10] For any ϵ, if there exists a $\delta > 0$ such that

$$|f^\sigma(t) - f(s) - f^\Delta(t)(\sigma(t) - s)| \le \epsilon|\sigma(t) - s| \quad \text{for all} \quad s \in (t - \delta, t + \delta) \cap \mathbb{T},$$

then f is called *delta (or Hilger) differentiable* on $\mathbb{T}^\kappa$ and f^Δ is called *delta derivative* of f.

Theorem 1 [1, Theorem 1.16] *Let $f : \mathbb{T} \to \mathbb{R}$ be a function with $t \in \mathbb{T}^\kappa$. Then*

a. If f is differentiable at t, f is continuous at t.

b. If f is continuous at t and t is right-scattered, then f is differentiable at t and

$$f^\Delta(t) = \frac{f^\sigma(t) - f(t)}{\mu(t)}.$$

c. If t is right dense, then f is differentiable at t iff

$$f^\Delta(t) = \lim_{s \to t} \frac{f(t) - f(s)}{t - s}$$

exists as a finite number.

d. If f is differentiable at t, then

$$f^{\sigma}(t) = f(t) + \mu(t) f^{\Delta}(t). \tag{2}$$

Note that $f^{\Delta} = f'$ (usual derivative) when $\mathbb{T} = \mathbb{R}$, while $f^{\Delta} = \Delta f$ (forward difference operator) if $\mathbb{T} = \mathbb{Z}$. The sum, product and quotient rules on time scales are introduced as follows:

Theorem 2 [1, Theorem 1.20] *For $t \in \mathbb{T}^{\kappa}$, suppose that $f, g : \mathbb{T} \to \mathbb{R}$ are differentiable at t. Then*

a. The sum $f + g : \mathbb{T} \to \mathbb{R}$ is differentiable at t with

$$(f+g)^{\Delta}(t) = f^{\Delta}(t) + g^{\Delta}(t).$$

b. If the product $fg : \mathbb{T} \to \mathbb{R}$ is differentiable at t, then

$$(fg)^{\Delta}(t) = f^{\Delta}(t)g(t) + f^{\sigma}(t)g^{\Delta}(t) = f(t)g^{\Delta}(t) + f^{\Delta}(t)g^{\sigma}(t).$$

c. If $g(t)g^{\sigma}(t) \neq 0$, then $\frac{f}{g}$ is differentiable at t with

$$\left(\frac{f}{g}\right)^{\Delta}(t) = \frac{f^{\Delta}(t)g(t) - f(t)g^{\Delta}(t)}{g(t)g^{\sigma}(t)}.$$

Let $f : \mathbb{T} \to \mathbb{R}$. Then f is said to be rd-continuous, if it is continuous at right dense points in $\mathbb{T}$ and its left sided limits exist as a finite number at left dense points in $\mathbb{T}$. Note also that every rd-continuous function has an antiderivative. Moreover, F given by

$$F(t) = \int_{t_0}^{t} f(s)\Delta s \quad \text{for} \quad t \in \mathbb{T}$$

is called an antiderivative of f, see [1, Theorem 1.74].

Stability and Lyapunov Function

In this section, the stability concept on time scales are introduced. The notations $\mathbf{x} = \mathbf{x}(t)$ and $\dot{V}(t, \mathbf{x}) = [V(\mathbf{x}(t))]^{\Delta}$ are used throughout. Let $V : \mathbb{R}^3 \to [0, \infty)$ be a Lyapunov function defined as

$$V(\mathbf{x}) = V_1(x_1) + V_2(x_2) + V_3(x_3),$$

where each $V_i : \mathbb{R} \to \mathbb{R}$ is continuously differentiable. There are several methods for computing $\dot{V}$ such as chain and product rules. Even though the system is autonomous and $V = V(\mathbf{x})$, $\dot{V}$ depends on t since the graininess function of $\mathbb{T}$ is not always constant. For the sake of the article, we give the following definitions, see [4].

Definition 3 A function $\phi : [0, r] \to [0, \infty)$ is called a *class of $\mathcal{K}$* if it is continuous, well-defined, and strictly increasing on $[0, r]$, where $\phi(0) = 0$.

Consider the first-order system of dynamic equations

$$\mathbf{x}^{\Delta} = f(t, \mathbf{x}), \quad t \geq t_0, \quad \mathbf{x} \in \mathcal{D} \subset \mathbb{R}^n, \tag{3}$$

where $\mathcal{D}$ is a compact set. Here, the function f is continuous and the existence of solutions to (3) subject to $\mathbf{x}(t_0) = \mathbf{x_0}$ is guaranteed with the uniqueness. Further, we assume $f(t, \mathbf{0}) = \mathbf{0} \in \mathcal{D}$ for all $t \in \mathbb{T}, t \geq t_0$ so that $\mathbf{x} = \mathbf{0}$ is a solution of equation (3). For the sake of the paper, we use the notation $\mathbf{x}(t, \mathbf{x_0}, t_0)$ for the solution with initial values $\mathbf{x_0} := \mathbf{x}(t_0) \in \mathcal{D}$ for $t_0 \in \mathbb{T}$.

Definition 4 The equilibrium solution $\mathbf{x} = \mathbf{0}$ of system (3) is called *stable* if there exists a function $\phi \in \mathcal{K}$ such that

$$|\mathbf{x}(t, \mathbf{x_0}, t_0)| \leq \phi(|\mathbf{x_0}|) \quad \text{for all} \quad t \in \mathbb{T}, \quad t \geq t_0.$$

On the other hand, $\mathbf{x = 0}$ is said to be *asymptotically stable* if it is stable and if there exists a positive $c \in \mathbb{R}$ such that

$$\lim_{t \to \infty} \mathbf{x}(t, \mathbf{x_0}, t_0) = \mathbf{0},$$

whenever $|\mathbf{x_0}| < c$.

Definition 5 Let $P : \mathbb{R}^n \to \mathbb{R}$ be a continuous function with $P(\mathbf{0}) = 0$. Then P is said to be *positive definite* (negative definite) on $\mathcal{D}$ if there exists a function $\phi \in \mathcal{K}$, such that $\phi(|\mathbf{x}|) \leq P(\mathbf{x})(\phi(|\mathbf{x}|) \leq -P(\mathbf{x}))$ for $\mathbf{x} \in \mathcal{D}$. On the other hand, P is called *positive semidefinite* (*negative semidefinite*) on $\mathcal{D}$ if $P(\mathbf{x}) \geq 0(P(\mathbf{x}) \leq 0)$ for all $\mathbf{x} \in \mathcal{D}$.

Definition 6 Suppose that the function $Q : [t_0, \infty) \times \mathbb{R}^n \to \mathbb{R}$ is continuous with $Q(t, 0) = 0$. Then Q is said to be *positive definite* (*negative definite*) on $t_0 \times \mathcal{D}$ if there exists a function $\phi \in \mathcal{K}$ such that $\phi(|\mathbf{x}|) \leq Q(t, x)$ $(\phi(|\mathbf{x}|) \leq -Q(t, x))$ for all $t \in \mathbb{T}$, $t \geq t_0$ and $\mathbf{x} \in \mathcal{D}$. On the other hand, Q is called *positive semidefinite* (*negative semidefinite*) on $t_0 \times \mathcal{D}$ if $Q(t, \mathbf{x}) \geq 0$ $(Q(t, x) \leq 0)$ for all $t \in \mathbb{T}$, $t \geq t_0$ and $\mathbf{x} \in \mathcal{D}$.

The following theorems give the stability and asymptotic stability criteria for equation (3), respectively.

Theorem 3 [4, Theorem 1] *The equilibrium solution* $\mathbf{x} = \mathbf{0}$ *of* (3) *is stable if there is a continuously differentiable positive definite function* V *in a neighborhood of zero with* $\dot{V}(t, x)$ *is negative semidefinite.*

Theorem 4 [4, Corollary 1] *The equilibrium solution* $\mathbf{x} = \mathbf{0}$ *of* (3) *is asymptotically stable, provided that there exists a continuously differentiable, positive definite function* V *in a neighborhood of zero with* $\dot{V}(t, \mathbf{x})$ *negative definite.*

Invariance Set Theorem

In this section, we provide an invariance principle for solutions of equation (3) by using Lyapunov function, see [4]. Let $a, b \in \mathbb{T}$, $\infty \leq a < 0 < b \leq \infty$ and $\phi : (a, b) \to E$, where E is an open set in $\mathbb{R}^n$. Define $T(t) : \mathbb{R}^n \to \mathbb{R}^n$, such that

$$x(t, \mathbf{x_0}) = T(t)\mathbf{x_0},$$
$$x(0, \mathbf{x_0}) = \mathbf{x_0} = T(0)\mathbf{x_0}, \quad T(0) = I,$$

where I is the identity mapping. More details for the properties of T are given in [4, Section 5].

Definition 7 Suppose that $t_n \in (a, b)$ is a sequence of points such that $t_n \to b$ $(t_n \to a)$ as $n \to \infty$ and $\lim_{n\to\infty} \phi(t_n) = p$. Then p is called a *positive (negative) limit point* of ϕ.

Definition 8 A solution $x(t, \mathbf{x_0})$ is called *positively* (*negatively*) *precompact* relative to E, provided that it is bounded for all $t \in [0, b(\mathbf{x_0}))(t \in (a(\mathbf{x_0}), 0])$ and has no limit points on the boundary of E.

For the convenience, the followings are introduced:

1. $H = \{\mathbf{x} : \ \dot{V}(\mathbf{x}) = 0, \ \mathbf{x} \in \bar{E} \cap E\}$.
2. M is the largest invariant set in H.
3. $V^{-1}(c) = \{\mathbf{x} \in \bar{E} \cap E : \ V(\mathbf{x}) = c\}$, for some finite c.

Theorem 5 *Suppose that V is a Lyapunov function of equation (3) on E and $x(t, \mathbf{x_0})$ is a solution of equation (3) that remains in E for all $t \in [0, b(x_0))$. If $x(t, \mathbf{x_0})$ is precompact, then $x(t, \mathbf{x_0}) \to M \cap V^{-1}(c)$ for some c.*

Stability Results

This section provides us the stability of equilibrium point $\mathbf{x} = \mathbf{0}$ of system (1), where $\mathbf{x} = (\alpha, \beta, \gamma)$. In order to calculate the derivative of Lyapunov function, we apply equation (2), which is known as the simple useful formula in the literature. Therefore, the forward jump operator σ is needed on α of the right hand side of system (1). One can see that this coincides with the continuous case since $\sigma(t) = t$ when $\mathbb{T} = \mathbb{R}$.

Stability of the Zero Solution

In order to achieve our goal, we first modify the controllers introduced in [5] by replacing $\sin\beta$ with $\sin\beta^\sigma$ in system (1). This follows from the fact that $\frac{\sin\beta^\sigma}{\beta^\sigma}$ is bounded but $\frac{\sin\beta}{\beta^\sigma}$ may not.

Theorem 6 *Consider system (1) with the feedback controller*

$$\begin{cases} v(t) = k_1\alpha^\sigma(t)\cos\beta(t) \\ w(t) = k_2\beta^\sigma(t) + k_1\frac{\sin\beta^\sigma(t)\cos\beta(t)}{\beta^\sigma(t)}\left(\beta^\sigma(t) + k_3\gamma^\sigma(t)\right), \end{cases} \tag{4}$$

where $k_1, k_2, k_3 > 0$. Then the equilibrium solution $\mathbf{x} = \mathbf{0}$ of system (1) is stable.

Proof Consider the Lyapunov function

$$V(\mathbf{x}) = \frac{1}{2}(\alpha^2 + \beta^2 + k_3\gamma^2), \tag{5}$$

which is continuously differentiable and positive definite in a neighborhood of zero. Taking the derivative of (5) by using the product rule for derivatives and (2) give us

$$\begin{aligned} \dot{V}(\mathbf{x}) &= \frac{1}{2}\left(\alpha^\Delta(\alpha + \alpha^\sigma) + \beta^\Delta(\beta + \beta^\sigma) + k_3\gamma^\Delta(\gamma + \gamma^\sigma)\right) \\ &= \frac{1}{2}\left(\alpha^\Delta(2\alpha^\sigma - \mu\alpha^\Delta) + \beta^\Delta(2\beta^\sigma - \mu\beta^\Delta) + k_3\gamma^\Delta(2\gamma^\sigma - \mu\gamma^\Delta)\right) \\ &\le \alpha^\Delta\alpha^\sigma + \beta^\Delta\beta^\sigma + k_3\gamma^\Delta\gamma^\sigma \\ &= -v\alpha^\sigma\cos\beta + \beta^\sigma\left(\frac{\sin\beta^\sigma}{\alpha^\sigma}v - w\right) + k_3\gamma^\sigma\frac{\sin\beta^\sigma}{\alpha^\sigma}v, \end{aligned}$$

where we use the equations of system (1). Now, by using controllers (4), we have

$$\dot{V}(\mathbf{x}) \le -k_1(\alpha^\sigma)^2\cos^2\beta - k_2(\beta^\sigma)^2 \le 0, \tag{6}$$

which implies that $\dot{V}$ is negative semidefinite and so Theorem 3 completes the proof. □

Next question that arises is whether we could show the asymptotic stability of the equilibrium solution $\mathbf{x} = \mathbf{0}$, i.e., $(\alpha, \beta, \gamma) \rightarrow (0, 0, 0)$. The following theorem shows that we only achieve that α and β tend to zero via Theorem 5, which is known as La Salle Invariance set theorem in the literature.

Theorem 7 *Consider system (1) with the feedback controllers (4) and let the solution (α, β, γ) of system (1) be precompact, then $(\alpha(t), \beta(t), \gamma(t)) \rightarrow (0, 0, c_1)$ as $t \rightarrow \infty$, where c_1 is a finite number.*

Proof Consider the Lyapunov function (5). It is shown that $\dot{V} \leq 0$ in the proof of Theorem 6, which implies that the Lyapunov function V is bounded. Therefore, the solution (α, β, γ) of system (1) is also bounded. Then by Theorem 5, we conclude that $\dot{V}$ tends to zero. Therefore, using equation (6) leads us to that α and β also tend to zero. Therefore, this completes the assertion. □

In Theorem 7, the asymptotic stability of the equilibrium solution $\mathbf{x} = \mathbf{0}$ is not shown on general time scales because $\dot{V}$ is negative semidefinite but not negative definite. Nevertheless, when $\mathbb{T} = \mathbb{R}$ (i.e., $\mu(t) = 0$ for all $t \in \mathbb{R}$), the asymptotic stability of the equilibrium solution $\mathbf{x} = \mathbf{0}$ is shown by Luca, Oriola and Vendittelli, see [5].

For general time scales, if we could show

$$\beta^{\Delta} = k_1 \sin \beta^{\sigma} \cos \beta - k_2 \beta^{\sigma} - k_1 \frac{\sin \beta^{\sigma} \cos \beta}{\beta^{\sigma}} \left(\beta^{\sigma} + k_3 \gamma^{\sigma} \right) \rightarrow 0 \tag{7}$$

as $\beta \rightarrow 0$, then we could unify and extend the asymptotic stability result of the system for general time scales. Nevertheless, one can consider the following discrete time scale, see [1], in order to show the asymptotic stability of $\mathbf{x} = \mathbf{0}$ of system (1).

Example 1 [1, Example 1.46] Let ν_n, $n \in \mathbb{N}_0$ be a sequence of real numbers with $\nu_n > 0$ for all $n \in \mathbb{N}$ such that ν_n tends to a nonzero finite limit as $n \rightarrow \infty$. Let

$$t_n = \sum_{k=0}^{n-1} \nu_k.$$

Consider the time scale $\mathbb{T} = \{t_n : n \in \mathbb{N}\}$, provided $\sum_{k=0}^{\infty} \nu_k = \infty$. Note that $\sigma(t_n) = t_{n+1}$, $\mu(t_n) = \nu_n$ for all $n \in \mathbb{N}$ and recall that $\beta(t) \rightarrow 0$ as $t \rightarrow \infty$ by Theorem 7. Therefore,

$$\beta^{\Delta}(t_n) = \frac{\beta(t_{n+1}) - \beta(t_n)}{\nu_n} \rightarrow 0$$

as $n \rightarrow \infty$, which implies that $\gamma(t_n) \rightarrow 0$ by (7). In conclusion, on this time scale, it could be shown that (α, β, γ) can asymptotically be driven to $\mathbf{0}$.

Asymptotic Stability of the Zero Solution

In Sect. 3.1, the asymptotic stability of the equilibrium solution $\mathbf{x} = \mathbf{0}$ of system (1) cannot be shown by means of controllers (4) on a general time scale. On the other hand, this section proposes different controllers for asymptotic stability of the zero solution of system (1). Therefore, we have the following theorem.

Theorem 8 *System (1) can asymptotically be driven to the origin* (0, 0, 0) *with the feedback controllers*

$$\begin{cases} v(t) = \alpha^\sigma(t)(k_1 \cos\beta(t) - k_2\gamma^\sigma(t)\sin\beta^\sigma(t)) \\ w(t) = k_1 \sin\beta^\sigma(t)\cos\beta(t) + k_3\beta^\sigma(t) - k_2\gamma^\sigma(t)\sin^2\beta^\sigma(t) \\ \quad + \frac{1}{\beta^\sigma(t)}\sin\beta^\sigma(t)\cos\beta(t)\gamma^\sigma(t)\left((k_2\alpha^\sigma(t))^2 + k_1k_4\right), \end{cases} \tag{8}$$

where $k_i > 0$ *for* $i = 1, \ldots, 4$.

Proof Consider the Lyapunov function

$$V(\mathbf{x}) = \frac{1}{2}(\alpha^2 + \beta^2 + k_4\gamma^2), \tag{9}$$

which is continuously differentiable and positive definite in a neighborhood of zero. By the similar discussion as in Theorem 6, we have

$$\dot{V}(\mathbf{x}) \le -v\alpha^\sigma\cos\beta + \beta^\sigma\left(\frac{\sin\beta^\sigma}{\alpha^\sigma}v - w\right) + \gamma^\sigma\frac{\sin\beta^\sigma}{\alpha^\sigma}v.$$

Next, by using controllers (8), we have

$$\dot{V}(\mathbf{x}) \le -k_1(\alpha^\sigma)^2\cos^2\beta - k_3(\beta^\sigma)^2 - k_2k_4(\gamma^\sigma)^2\sin^2\beta^\sigma \le 0, \tag{10}$$

which implies that $\dot{V}$ is negative definite. Then Theorem 4 completes the proof. □

Simulation Results

In this section, we provide the simulations that help us observe the main results visually when we reduce system (1) into a specific time scale.

Example 2 Consider system (1) with the feedback controllers (8) when $\mathbb{T} = \mathbb{R}$ with the initial values $\alpha(0) = 1400$, $\beta(0) = \frac{2\pi}{3}$, $\gamma(0) = -\frac{\pi}{2}$. The constants in the control inputs are $k_1 = 0.1$, $k_2 = 0.1$, $k_3 = 0.8$ and $k_4 = 0.8$. The robot is initiated from an arbitrary position and move to a desired position. The following figures show that we can take the robot to our desired point Figs. 2, 3, 4.

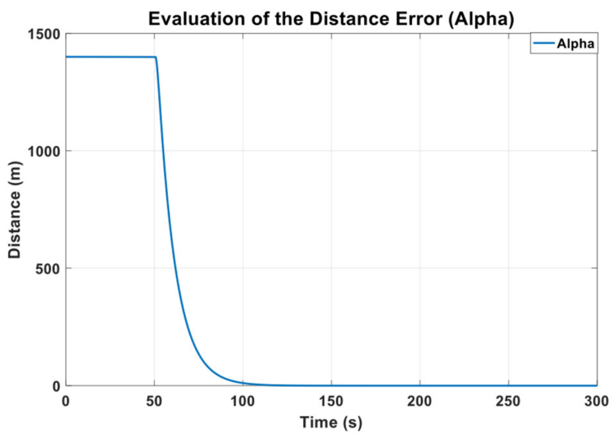

Fig. 2 Evaluation of the distance error α

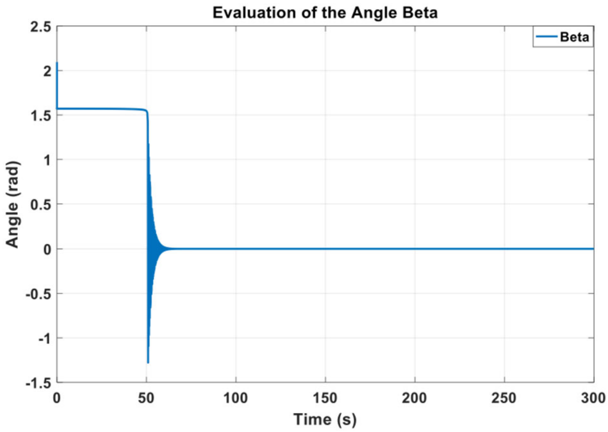

Fig. 3 Evaluation of the angle β

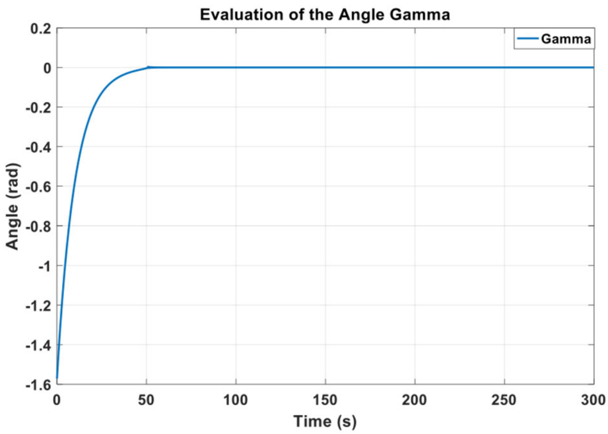

Fig. 4 Evaluation of the angle γ

Conclusion

In this paper, the stability results of system (1) are shown by proposing two different controllers. Since the asymptotic stability of system (1) cannot be unified on general time scales by using the controllers (4), motivated from [5], we only give the stability of system (1) in Sect. 3.1. On the other hand, this leads us to provide the new controllers (8) and we have successfully obtained the criteria for asymptotic stability of system (1) in Sect. 3.2. Therefore, we have generalized the results in continuous case to the time scale theory which enable to reduce the computational cost when simulating the main results. Another thing that one should observe is that Luca et all used Barbalat Lemma to show the asymptotic stability of system (1) when $\mathbb{T} = \mathbb{R}$ in [5]. However, since we do not have the time scale version of Barbalat Lemma, we had to provide a new controller to show the asymptotic stability for general time scales.

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