Math 15, Exam 1, Jan 27, 2005

Student's Name:	KEY
Instructor's Name:	
Recitation Time:	

INSTRUCTIONS

Calculators may be used on this exam.
However, if a problem does not say to use a calculator, then you must show your work in order to receive credit.

1. Be sure to print your name and your instructor's name in the space provided at the top of this page.
2. Work all problems. Show all work. Full credit will be given only if work is shown which fully justifies your answer.
3. There will be sufficient space under each problem in which to show your work.
4. Circle, box, or underline each final answer. *All final answers must be simplified!*
5. This exam has 4 sheets of paper (front and back). Do not remove the staple! There are 100 points. Each of the ten problems is 10 points.
6. **Turn off your cell phone if you have one with you.**

DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

#	1	2a	2b	3	4a	4b	5	6a	6b	7	Σ
P											
Σ	10	10	10	10	10	10	10	10	10	10	100

1. Find $(f^{-1})'(4)$, where $f(x) = x^5 + x^3 + 2x$.

$f'(x) = 5x^4 + 3x^2 + 2 > 0 \Rightarrow f$ is 1-1.

$f^{-1}(4) = x \Leftrightarrow f(x) = 4 \Rightarrow x = 1$

$$(f^{-1})'(4) = \frac{1}{f'(f^{-1}(4))} = \frac{1}{f'(1)} = \frac{1}{5+3+2} = \boxed{\frac{1}{10}}$$

2. Find the derivative of the following functions:

(a) $F(t) = e^{t\sin(3t)}$

$$F'(t) = e^{t\sin 3t} \cdot (t\sin 3t)'$$

$$= \boxed{e^{t\sin 3t}(\sin 3t + 3t\cos 3t)}$$

(b) $H(x) = (1 + x^2)\arctan x$

$$H'(x) = 2x \cdot \arctan x + (1+x^2)\frac{1}{1+x^2}$$

$$= \boxed{2x\arctan x + 1}$$

3. Find the tangent line to $y = \frac{e^x}{x}$ at the point $(1, e)$.

$$y' = \frac{e^x \cdot x - e^x}{x^2} = \frac{e^x (x-1)}{x^2} \Rightarrow y'\Big|_{(1,e)} = 0$$

$$y - e = 0(x-1) \Rightarrow \boxed{y = e}$$

4. If a bacteria population starts with 100 bacteria and doubles every three hours, then the number of bacteria after t hours is $f(t) = 100 \cdot 2^{t/3}$.

(a) Find the inverse of this function and explain its meaning.

$$y = 100\, 2^{t/3} \Rightarrow \frac{y}{100} = 2^{t/3}$$

$$\Rightarrow \log_2 \frac{y}{100} = \log_2 2^{t/3}$$

$$\Rightarrow \log_2 \frac{y}{100} = t/3$$

$$\Rightarrow \boxed{t = 3 \log_2 \frac{y}{100}}$$

Its meaning:

The function tells how long it will take to obtain y bacteria.

(b) When will the population reach 50,000?

$$f^{-1}(+) = 3 \log_2 \frac{50,0\cancel{00}}{1\cancel{00}}$$

$$= \boxed{3 \log_2 500}$$

5. Use logarithmic differentiation to differentiate

$$y = (2x+1)^5(x^4-8)^6.$$

$$\ln y = \ln (2x+1)^5 (x^4-8)^6$$

$$\ln y = 5 \ln (2x+1) + 6 \ln (x^4-8)$$

$$\frac{y'}{y} = 5 \frac{2}{2x+1} + 6 \frac{4x^3}{x^4-8}$$

$$y' = y\left(\frac{10}{2x+1} + \frac{24x^3}{x^4-8} \right)$$

$$y' = (2x+1)^5 (x^4-8)^6 \left[\frac{10}{2x+1} + \frac{24x^3}{x^4-8} \right]$$

$$y' = \boxed{10(2x+1)^4(x^4-8)^6 + 24x^3(2x+1)^5(x^4-8)^5}$$

6. Evaluate the following integrals:

(a) $\displaystyle\int_2^6 \frac{2}{x}dx = 2\ln x \Big|_2^6$

$= 2\left[\ln 6 - \ln 2\right]$

$= 2\ln 3$

$= \boxed{\ln 9}$

(b) $\displaystyle\int \frac{\sin x}{1+\cos^2 x}dx = -\int \frac{du}{1+u^2}$

$u = \cos x$

$du = -\sin x\,dx$

$= -\arctan u + C$

$= \boxed{-\arctan(\cos x) + C}$

7. Find $\displaystyle\lim_{t\to 0}\frac{5^t - 8^t}{t}$.

$$\underset{\left(\frac{0}{0}\right)}{=} \lim_{t\to 0} \frac{5^t \ln 5 - 8^t \cdot \ln 8}{1}$$

$$= \ln 5 - \ln 8$$

$$= \boxed{\ln \frac{5}{8}}$$