
NOW PLAYING
ALGEBRA II
THE SEQUEL
"TWO THUMBS UP!"
SICKEL & ELBERT
22B
MRS. BELL
BLAIR

EXAM 1

Jan 27 200

5 — 6 pm

CE 125

MLC
REVIEW

Jan 26, 200

Mc-Nutt 20

7-9 pm

EXAM I (7.1, 7.2, 7.3, 7.4, 7.5, 7.7)

Sec 7.1 Inverse Functions

- 1-1 functions
  - $f^{-1}(x) = y \iff f(y) = x$
  - $f' \Rightarrow f \downarrow$ or $\uparrow$
  - HLT : graph of f

- $f : A \to B$ 1-1 $\Rightarrow f^{-1}$ exists.

- To find f^{-1} : (a) $y = f(x)$ (b) Solve for x
  (c) Interchange x and y
  (d) The resulting eqn. $= f^{-1}$.

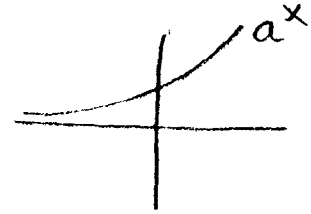
- To calculate $(f^{-1})'(a)$: $\quad (f^{-1})'(a) = \dfrac{1}{f'(\underbrace{f^{-1}(a)}_{\text{use the defn. of } f})}$.

- To draw the graph of f^{-1}: Reflection about the line $y = x$.

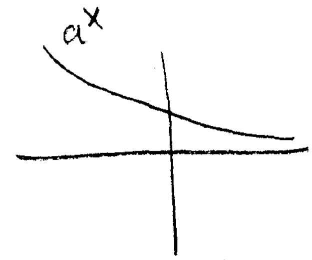
Sec. 7.2 Exponential Functions and Their Derivatives

$$f(x) = a^x, \quad a > 0$$

$$a^x : \mathbb{R} \to (0, +\infty)$$



$a > 1$



$a < 1.$

1)

- $a^x > 0$ for all x
- $a^{x+y} = a^x \cdot a^y$
- $(a^x)^y = a^{xy}$
- $(ab)^x = a^x b^x$

(2) • $a > 1 \Rightarrow a^x \xrightarrow[x \to +\infty]{} \infty, \quad a^x \xrightarrow[x \to -\infty]{} 0$

• $0 < a < 1 \Rightarrow a^x \xrightarrow[x \to +\infty]{} 0, \quad a^x \xrightarrow[x \to -\infty]{} \infty$

(3) • $(a^x)' = a^x \ln a$

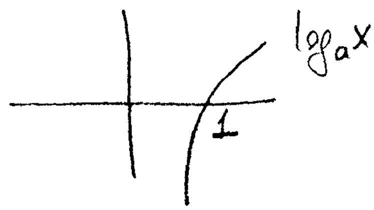
• $(e^x)' = e^x$

• $\left(e^{g(x)}\right)' = e^{g(x)} \cdot g'(x)$ CHAIN RULE

• $\int e^x \, dx = e^x + C$

Sec 7.3 Logarithmic Functions

- $a^x \leftrightarrow 1-1$
- $a^x : \mathbb{R} \to (0, +\infty)$, $(a^x)^{-1} = \log_a x : (0, +\infty) \to \mathbb{R}$
- $a > 1$



- $\log_a x \xrightarrow[x \to +\infty]{} \infty$, $\log_a x \xrightarrow[x \to 0^+]{} -\infty$

- $$\left.\begin{aligned} \log_a(xy) &= \log_a x + \log_a y \\ \log_a\left(\frac{x}{y}\right) &= \log_a x - \log_a y \\ \log_a x^r &= r \log_a x \end{aligned}\right\}$$:) for log. diff

- $e^x \longleftrightarrow \underbrace{(e^x)^{-1}}_{\text{inverse}} = \ln x \quad 2 < e < 3$

- $\log_a x = \dfrac{\ln x}{\ln a}$

- Eqn. of the tangent to a curve.

Sec. 7.4 Derivatives of Logarithmic Functions

- $(\ln x)' = \frac{1}{x}$
- $(\ln(g(x)))' = \frac{g'(x)}{g(x)}$ Chain Rule.
- $(\ln|x|)' = \frac{1}{x}$
- $\int \frac{1}{x}\,dx = \ln|x| + c$
- $\int \tan x\,dx = \ln|\sec x| + c$
- $(\log_a x)' = \frac{1}{\ln a}\,\frac{1}{x}$
- $(\log_a g(x))' = \frac{1}{\ln a} \cdot \frac{g'(x)}{g(x)}$
- $\int a^x\,dx = \frac{1}{\ln a}\,a^x + c$
- $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$, $n \neq -1$
- Logarithmic Differentiation $\frac{d}{dx}\left(f(x)^{g(x)}\right)$

  (a) Take logarithms of both sides of $y = f(x)$

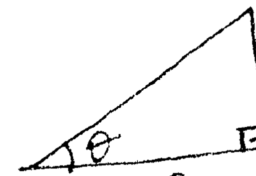
  (b) Differentiate implicitly with respect to x.

  (c) Solve the resulting eqn. for y'

  OR write T function as an exponentio function

Sec. 7.5 Inverse Trigonometric Functions

- $\sin x : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1,1]$
- $\sin^{-1} x : [-1,1] \longrightarrow [-\pi/2, \pi/2]$
- $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
- $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + c$
- $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$ ★ $\boxed{\text{u-subs.t}}$

★ 

simplify cos(tan

- $\cos x : [0,\pi] \longrightarrow [-1,1]$

$\cos^{-1} x : [-1,1] \longrightarrow [0,\pi]$

$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$

- $\tan x : (-\pi/2, \pi/2) \longrightarrow \mathbb{R}$

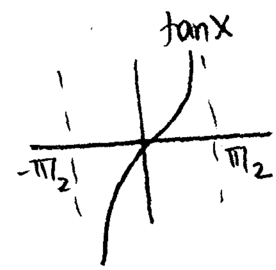
$\tan^{-1} x : \mathbb{R} \longrightarrow (-\pi/2, \pi/2)$

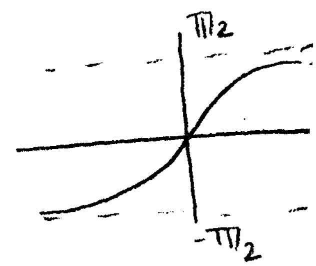
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

$\int \frac{dx}{1+x^2} = \tan^{-1} x + c$

$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c.$

For example LIM





$\tan^{-1} x \xrightarrow[x\to\infty]{} \pi/2$

$\tan^{-1} x \xrightarrow[x\to-\infty]{} -\pi/2$

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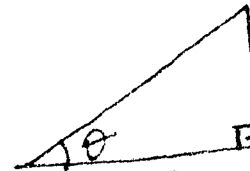
★ 

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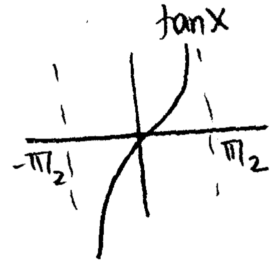

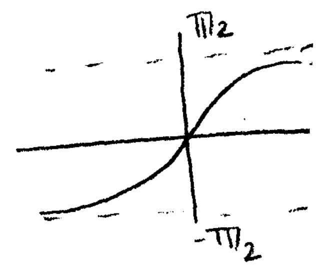

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Sec 7.7 Indeterminate Forms

$$\left.\begin{aligned}(fg)' &= f'g + fg' \\ \left(\frac{f}{g}\right)' &= \frac{f'g - g'f}{g^2}\end{aligned}\right\}$$

DO NOT CONFUSE with L'HOSPITAL RULE $\left(\frac{\infty}{\infty}\right)$ $\left(\frac{0}{0}\right)$

DO NOT USE IT IF YOU DO NOT NEED IT.

- $\lim \frac{f}{g} \underset{\left(\frac{0}{0}\right)\,\left(\frac{\infty}{\infty}\right)}{=} \lim \frac{f'}{g'}$

- $0 \cdot \infty \longrightarrow$ convert the product into a quotient

  $\frac{0}{0}$ / $\frac{\infty}{\infty}$ (LHR)

- $\infty - \infty \longrightarrow$ convert the difference into a quotient

  $\frac{\infty}{\infty}$ / $\frac{0}{0}$ (LHR)

  if it is getting ugly DO NOT USE LHR and simplify the quotient and the ...

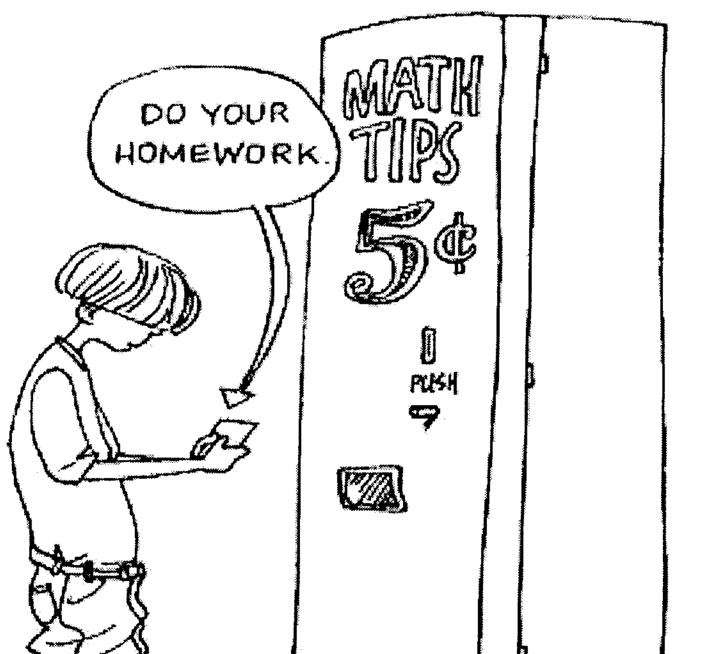
- $0^0, \infty^0, 1^\infty$ $\nearrow$ Use logarithm

  $\searrow$ Rewrite the funct. as an exp fun

$$y = f(x)^{g(x)} \longrightarrow \ln y = g(x)\ln(f(x)) \xrightarrow[\lim]{} a$$

$$\Rightarrow y \to e^a.$$

$$y = x^x \rightarrow y = e^{\ln x^x} = e^{x \ln x} \xrightarrow[\cancel{\lim}\ \infty]{} b$$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