EXAM 2 (8.1, 8.2, 8.3, 8.4, 8.5, 8.6, 8.8)

Sec. 8.1 Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Sec. 8.2 Trigonometric Integrals

$$\int \sin^m x \cos^n x \, dx$$

m or n (or both) ODD

- m odd $\Rightarrow$ $u = \cos x$
  $\sin^2 x = 1 - \cos^2 x$
- n odd $\Rightarrow$ $u = \sin x$
  $\cos^2 x = 1 - \sin^2 x$

m and n even

Use identities

- $\sin^2 x = \dfrac{1 - \cos(2x)}{2}$
- $\cos^2 x = \dfrac{1 + \cos(2x)}{2}$
- Integration by parts

$\int \sin(mx)\cos(nx)\,dx$ $\quad$ $\int \sin(mx)\sin(nx)\,dx$

$\int \cos(mx)\cos(nx)\,dx$

- $\sin A \cos B = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$
- $\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$
- $\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$.

$$\int \sec^m x \tan^n x \, dx$$

n odd $\Rightarrow$ $u = \sec x$

$\tan^2 x = \sec^2 x - 1$

m even $\Rightarrow$ $u = \tan x$

$\sec^2 x = 1 + \tan^2 x$

OTHERWISE ; USE IDENTITIES , INTEGRATON BY PARTS

$$\int \tan x \, dx = \ln|\sec x| + C$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

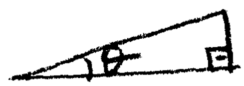
Sec 8.3 Trigonometric Substitution.

$\sqrt{a^2 - u^2} \Rightarrow u = a\sin\theta$, $\quad 1 - \sin^2\theta = \cos^2\theta$

$\sqrt{a^2 + u^2} \Rightarrow u = a\tan\theta$, $\quad 1 + \tan^2\theta = \sec^2\theta$

$\sqrt{u^2 - a^2} \Rightarrow u = a\sec\theta$, $\quad \sec^2\theta - 1 = \tan^2\theta$

* Use a right triangle



* Find the area of the region

Sec 8.4 Integration of Rational Functions by Partial Fractions

$$f(x) = \frac{P(x)}{Q(x)}$$

* $\deg P(x) > \deg Q(x) \Rightarrow$ divide $P(x)$ by $Q(x)$.

Case 1 : $Q(x)$ is a product of distinct linear factors

$$\frac{x^2 + 2x - 1}{x(2x-1)(x+2)}$$

Case 2 : $Q(x)$ is a product of linear factors, some of which are repeated.

$$\frac{x^3 - x + 1}{x^2(x-1)^3}$$

Case 3 : $Q(x)$ contains irreducible quadratic factors, none of which is repeated.

$$\frac{x}{(x-2)(x^2+1)(x^2+4)}$$

Case 4 : $Q(x)$ contains a repeated irreducible quadratic factor

$$\frac{x^3 + x^2 + 1}{x(x-1)(x^2+x+1)(x^2+1)^3}$$

Sec. 8.5 Strategy for Integration (Review)

Sec. 8.8 Improper Integrals

<u>Type 1</u> Integral over an infinite interval

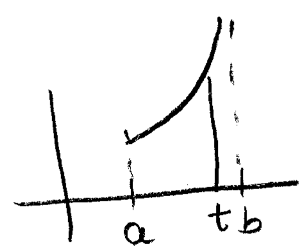
<u>Type 2</u> Integral over a finite interval, in where f is discontinuous.

$$\int_a^{\infty} f(x)\,dx = \lim_{t\to\infty} \int_a^t f(x)\,dx$$

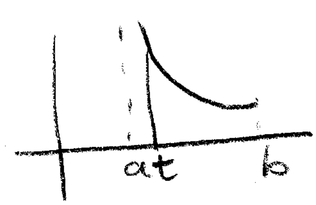
$$\int_{-\infty}^{a} f(x)\,dx = \lim_{t\to-\infty} \int_t^b f(x)\,dx$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{a} f(x)\,dx + \int_a^{\infty} f(x)\,dx$$

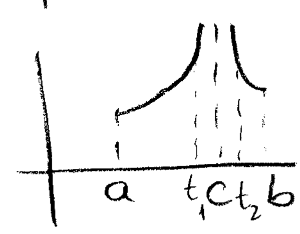
$$\int_a^b f(x)\,dx = \lim_{t\to b^-} \int_a^t f(x)\,dx$$



$$\int_a^b f(x)\,dx = \lim_{t\to a^+} \int_t^b f(x)\,dx$$



$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$



☆☆ APPLICATIONS ☆☆.