







# **Abstract Book**

Edited by

Martin Bohner, Okay Çelebi, and Mehmet Ünal

Beşiktaş Campus of Bahçeşehir University

Istanbul, Turkey

27 June - 1 July 2005



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#### **Preface**

#### Dear Colleague:

It is our great pride and pleasure to offer our warmest greetings to you, the participants in our international workshop on "Dynamic equations on Time Scales" at the Beşiktaş campus of Bahçeşehir University in Istanbul, Turkey. This workshop is sponsored by **TÜBİTAK** (Scientific and Technical Research Council of Turkey), **Bahçeşehir University**, and the **Turkish Ministry of Culture and Tourism**.



The purpose of this workshop is to provide a platform of discussion and to share information among international scientists related to the subject. The meeting will take place in the form of keynote addresses, group discussions, and paper presentations.

In addition to attending the workshop's exciting sessions, we encourage each of the participants to take advantage of our historic city of Istanbul, which is the cradle of many civilizations, to share beauty and scientific knowledge. Last but not least we want to extend our best wishes to all of the workshop participants and to its Scientific and Organizing Committee members.

Sincerely,

Dr. Mehmet Ünal (Vice Dean)

Chair of Organizing Committee

University of Bahçeşehir, Faculty of Arts & Sciences

TR-34538 Bahçeşehir/Istanbul, Turkey

Workshop web site: http://www.fed.bahcesehir.edu.tr/workshop

# Organizers









# Yılmaz Akyıldız

Bosporus University Istanbul, Turkey

Martin Bohner University of Missouri–Rolla Rolla, MO, USA

**Okay Çelebi** Middle East Technical University Ankara, Turkey

> **Mehmet Ünal** Bahçeşehir University Istanbul, Turkey

# Scientific Committee





















#### Ravi Agarwal

Florida Institute of Technology Melbourne, FL, USA

#### Elvan Akın-Bohner

University of Missouri–Rolla Rolla, MO, USA

#### Martin Bohner

University of Missouri–Rolla Rolla, MO, USA

#### Okay Çelebi

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Ankara, Turkey

#### Lynn Erbe

University of Nebraska–Lincoln Lincoln, NE, USA

#### Gusein Guseinov

Atılım University Ankara, Turkey

#### Johnny Henderson

Baylor University Waco, TX, USA

#### Allan Peterson

University of Nebraska–Lincoln Lincoln, NE, USA

#### Mehmet Ünal

Bahçeşehir University Istanbul, Turkey

#### Ağacık Zafer

 $\label{eq:middle} \mbox{Middle East Technical University} \\ \mbox{Ankara, Turkey}$ 

# Plenary Speakers

















#### Martin Bohner

University of Missouri–Rolla Rolla, MO, USA

# Saber Elaydi Trinity University San Antonio, TX, USA

Gusein Guseinov Atılım University Ankara, Turkey

# Ondřej Došlý Masaryk University Brno, Czech Republic

**Stefan Hilger** Katholische Universität Eichstätt Eichstätt, Germany

# Donald Lutz San Diego State University San Diego, CA, USA

Andreas Ruffing
Technische Universität München
München, Germany

Christopher Tisdell University of New South Wales Sydney, Australia

#### About the Workshop

The study of Dynamic Equations on Time Scales, which goes back to its founder Stefan Hilger (1988), is an area of mathematics that has recently received a lot of attention. It has been created in order to unify the study of differential and difference equations. Many results concerning differential equations carry over quite easily to corresponding results for difference equations, while other results seem to be completely different from their continuous counterparts. The study of dynamic equations on time scales reveals such discrepancies, and helps to avoid proving results twice – once for differential equations and once again for difference equations. The general idea is to prove a result for a dynamic equation where the domain of the unknown function is a so-called time scale, which may be an arbitrary closed subset of the reals. This way results not only related to the set of real numbers or set of integers but those pertaining to more general time scales are obtained. The three most popular examples of calculus on time scales are differential calculus, difference calculus, and quantum calculus. Dynamic equations on a time scale have a large potential for applications such as in population dynamics. For example, it can model insect populations that are continuous while in season, die out in say winter, while their eggs are incubating or dormant, and then hatch in a new season, giving rise to a nonoverlapping population.

The recent books by Martin Bohner and Allan Peterson, "Dynamic Equations on Time Scales: An Introduction with Applications", published by Birkhäuser in 2001 and then "Advances in Dynamic Equations on Time Scales" by Birkhäuser in 2003, have attracted many researchers' attention to this area, and as a result "Dynamic Equations on Time Scales" is rapidly becoming an independent discipline by itself. At several occasions (AMS National, International, and Regional meetings, SIAM meetings, other national and international conferences) there have been many Special Sessions organized specifically on "Time Scales" in the recent years. Also, Martin Bohner and Allan Peterson were the main speakers of a summer conference on "Time Scales" on July 2002 in Laramie, WY in connection with RMMC which attracted various other mathematicians to this area. So there is a dynamic group of researchers working in this field who are striving to collaborate for new developments, and this workshop constitutes the best medium to bring these people together.





# THE FIRST

INTERNATIONAL WORKSHOP ON

DYNAMIC EQUATIONS ON TIME SCALES

in Memory of Bernd Aulbach

27 June - 1 July 2005

# In Memory of Bernd Aulbach



On January 14, 2005, Professor Bernd Aulbach suddenly and unexpectedly passed away at the age of 57 years. The Institute for Mathematics lost a valued colleague, a respected scientist, and a popular university teacher. We are all mourning his death.

Bernd Aulbach was born in Aschaffenburg on December 23, 1947, where he also went to school and graduated with the German high school diploma (Abitur) in 1967. Subsequently, he studied mathematics with a minor in physics in Würzburg. Early on, he discovered his interest in differential equations and consequently wrote his Master's thesis with Professor H. W. Knobloch on the topic: "The Domain of Attractivity of an Asymptotically Stable Solution for Nonautonomous Periodic Differential Equations". He graduated from Würzburg with the Master's degree (Diplom) in 1973 and remained in Würzburg as a scientific assistant to the chair, Professor Knobloch, with whom he completed the Ph.D. degree in 1976 by writing a thesis also on domains of attractivity of stable periodic solutions. He spent the academic year 1978/79 as a Visiting Assistant Professor at State University of New York in Albany. From 1983 until 1986 he had a fellowship from the "Volkswagenwerk Foundation" in order to study the project "Qualitative Analysis of Nonlinear Dynamic Systems by Means of Invariant Manifolds". In the context of this project, his State Doctoral thesis (Habilitationsschrift) "Continuous and Discrete Dynamics near Manifolds of Equilibria" emerged, which also appeared as Lecture Notes published by Springer in 1984. In August 1984, he became a lecturer (Privatdozent) at the University of Würzburg. Subsequently, he was awarded a highly competitive Heisenberg Scholarship from the DFG (German Research Society), which he used to finance a longer stay at the University of California in Berkeley in 1986/87. In the year 1987, he finally accepted a position at the University of Augsburg. In the year 1970, still as a student, he married Gudrun Nöll with whom he had one daughter and two sons; the children were born in the years 1971, 1976, and 1980.

As can be inferred from the quoted topic of his first research project, his interest was devoted to the qualitative theory of dynamical systems. And though he remained faithful to this area, it was natural that the emphasis of his work shifted. His main legacy is without doubt his steady effort to regard continuous and discrete dynamics from a common point of view. This begins 1984 with his Habilitationsschrift and culminates 2001 in his presidency of the "International Society of Difference Equations". He was thereby not only concerned with a new unification of different approaches, but also with bringing together scientists who work and perform research to a large extent separately in the areas of "differential equations" and "difference equations". This effort succeeded with the "Sixth International Conference on Difference Equations", which he organized in Augsburg in 2001. Until his death, he was considerably involved with the continuation of this conference series. Hand in hand with this, one must also recognize his activity as an editor of the "Journal of Difference Equations and Applications", to which, however, his editorial activity was not limited: The journals "Differential Equations and Dynamical Systems" and "Nonlinear Dynamics and Systems Theory" are to be mentioned as well. A crucial tool to build the bridge between continuous and discrete dynamics is the "measure chain calculus", and he was probably the first one to recognize its importance: He had already put the decisive basic principles together with his Ph.D. student S. Hilger in the late Eighties. A further emphasis of his research was nonautonomous systems, which do not produce dynamical systems in the classical sense. Here as well, he emphasized the importance of only measurable time dependence, i.e., he was concerned with a unified qualitative theory for both nonautonomous differential equations and nonautonomous difference equations. At all times he was particularly open for applications.

Together with Professor Colonius he also directed the working group "Dynamics and Control of Ordinary Differential Equations" within the scope of Augsburg's Graduate School (Graduiertenkolleg) in "Nonlinear Problems in Analysis, Geometry, and Physics". This Graduate School was granted to Augsburg in 1996 by the DFG, and from the very beginning, Bernd Aulbach was its speaker. It was also mainly due to his dedicated input that the Graduate School was extended twice and thus the maximum support duration was granted. Unfortunately he was unable to see his favorite project (as he admitted) through to its completion in this year 2005.

As a scientist, Bernd Aulbach was always active and successful, which is documented by his over 60 scientific publications. His text book on "Ordinary Differential Equations", which just appeared in its second edition, is much in demand.

Professor Aulbach was a highly gifted teacher and consequently was very popular among his students. This also resulted in a higher-than-average number of graduate

students. His student S. Siegmund received a renowned Emmy Noether Scholarship. Like his lectures, his presentations were characterized by extreme clarity. For this reason, he was invited to many national and international conferences, workshops, and colloquia. In order to participate, he did not hesitate to even take the longest journeys, e.g., to Vietnam, China, or India. As a colleague, Bernd Aulbach was extremely cooperative, and he never refused to serve in committees and advisory commissions. His balanced personality was esteemed, and his ideas advanced many a committee.

The Institute for Mathematics mourns the loss of a universally qualified colleague, a popular teacher, and a friend. We will remember him forever.

Hansjörg Kielhöfer, Augsburg (translated by Martin Bohner, Rolla)

# Schedule of the Talks

Table 1. Complete Program

Time	June 26	June 27	June 28	June 29	June 30	July 1
	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-9:30		Registration		Istanbul		
9:30-9:45		Opening		Tour		
9:45-10:45		Plenary Talk	Plenary Talk		Plenary Talk	Plenary Talk
		Hilger	Došlý	Istanbul	Guseinov	$\operatorname{Lutz}$
10:45-11:00		Refreshme	ent Break	Tour	Refreshm	ent Break
11:00-11:25		Garay	Hilscher		Anderson	Řehák
11:30-11:55		Rynne	Ünal	Istanbul	Lawrence	Pötzsche
12:00-12:25		Kaymakçalan	Akın-Bohner	Tour	Zafer	Ufuktepe
12:25-13:30		Lur	nch		Lu	nch
13:30-14:30		Plenary Talk	Plenary Talk	Istanbul	Plenary Talk	Plenary Talk
		Bohner	Elaydi	Tour	Ruffing	Tisdell
14:30-14:45	Registration	Refreshme	ent Break		Refreshment Break	
14:45-15:10		Cermak	András	Istanbul	Castillo	Bodine
		Batıt	Siafarikas	Tour	Yokuş	
15:15-15:40		Topal	Yantır		Stehlik	
	Registration	Kara	Došlá	Istanbul	Diblík	
15:40-16:00		Refreshme	ent Break	Tour	Break	
16:00-16:25		Şahiner	Langenius		Warth	
		Panahov	Wintz	Istanbul	$\operatorname{Sticka}$	
16:30-16:55	Registration	Silindir	Akın	Tour	Luo	
		Akça	Gaiko		Meiler	
Evening	Welcome	sightseeing	Dinner	Yacht	Farewell	
	Party	free time	at BU	Tour	Party	

Table 2. Schedule of Session A, Room A202  $\,$ 

	June 27, 2005	June 28, 2005	June 30, 2005	July 1, 2005	
	Monday	Tuesday	Thursday	Friday	
Chair	O. Çelebi	A. Zafer	B. Kaymakçalan	Y. Akyıldız	
09:45-10:45	Plenary Talk	Plenary Talk	Plenary Talk	Plenary Talk	
	Stefan	Ondřej	$\mathbf{Gusein}$	Donald	
	Hilger	Došlý	$\mathbf{Guseinov}$	Lutz	
	Germany (p. 25)	Czech Rep. (p. 22)	Turkey (p. 24)	USA (p. 28)	
10:45-11:00	Refreshment Break				
11:00-11:25	Barnabas	Roman	Douglas	Pavel	
	Garay	Hilscher	Anderson	Řehák	
	Hungary (p. 23)	Czech Rep. (p. 25)	USA (p. 18)	Czech Rep. (p. 30)	
11:30-11:55	Bryan	Mehmet	Bonita	Christian	
	Rynne	Ünal	Lawrence	Pötzsche	
	UK (p. 31)	Turkey (p. 34)	USA (p. 27)	USA (p. 29)	
12:00-12:25	Billûr	Elvan	Ağacık	Ünal	
	Kaymakçalan	Akın-Bohner	Zafer	Ufuktepe	
	USA (p. 26)	USA (p. 18)	Turkey (p. 36)	Turkey (p. 34)	
12:25-13:30		Lu	nch		

Chair	C. Tisdell	Y. Şahiner	B. Rynne	O. Çelebi
13:30-14:30	Plenary Talk	Plenary Talk	Plenary Talk	Plenary Talk
	Martin	Saber	Andreas	Christopher
	Bohner	Elaydi	Ruffing	$\mathbf{Tisdell}$
	USA (p. 20)	USA (p. 23)	Germany (p. 30)	Australia (p. 33)
14:30-14:45		Refreshm	ent Break	
14:45-15:10	Jan Cermak	Silárd András	José Castillo	Sigrun Bodine
	Czech Rep. (p. 21)	Romania (p. 18)	USA (p. 20)	USA (p. 19)
15:15-15:40	Serap Topal	Ahmet Yantır	Petr Stehlik	
	Turkey (p. 34)	Turkey (p. 35)	Czech Rep. (p. 33)	
15:40-16:00		Refreshment Break		
16:00-16:25	Yeter Şahiner	Peter Langenius	Howard Warth	
	Turkey (p. 31)	Sweden (p. 27)	USA (p. 35)	
16:30-16:55	Burcu Silindir	Ömer Akın	Hua Luo	
	Turkey (p. 32)	Turkey (p. 17)	China (p. 28)	

Table 3. Schedule of Session B, Room A205

	June 27, 2005	June 28, 2005	June 30, 2005
	Monday	Tuesday	Thursday
Chair	P. Siafarikas	J. Castillo	Z. Došlá
14:45-15:10	Özlem Batıt	Panos Siafarikas	Asif Yokuş
	Turkey (p. 19)	Greece (p. 32)	Turkey (p. 36)
15:15-15:40	Rukiye Kara	Zuzana Došlá	Josef Diblík
	Turkey (p. 26)	Czech Rep. (p. 22)	Czech Rep. (p. 21)
15:40-16:00		Refreshment Break	
16:00-16:25	Etibar Panahov	Nick Wintz	Wilhelm Sticka
	Turkey (p. 29)	USA (p. 35)	USA (p. 33)
16:30-16:55	Haydar Akça	Valery Gaiko	Maria Meiler
	Saudi Arabia (p. 17)	Belarus (p. 24)	Germany (p. 28)

#### Abstracts of the Talks

## Haydar Akça

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King Fahd University of Petroleum and Minerals, Department of Mathematical Science, P.O. Box 1071, Dhahran 31261, Saudi Arabia

Functional Differential Equations and Modelling with Some Special Type of Equations

Since Euler times, differential equations occupied a special place among mathematics equations, with deviating argument, integro-differential equations, impulsive arguments and other types. Certain classes of such equations become more and more significant in modern physics, biology and economics, and the application of functional differential equations penetrates all branches of modern sciences. Nowadays a distinguishing feature of the Functional Differential Equations (FDEs) is under consideration by numerous researchers and scientists. (Joint work with Shebadeh Mustafa Yaqoub, King Fahd University of Petroleum and Minerals.)

# Ömer Akın

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TOBB Economy and Technology University, Department of Mathematics, Faculty of Arts and Sciences, TR-06530 Söğütözü, Ankara, Turkey

An Approach to the Solution of Time-varying Linear Dynamic Systems with Two Point BVP on Time Scales

In this work we give some results on an approach to the solution of time varying linear dynamic systems with separated two-point boundary value problem on time scales (Joint work with Mohammad Dastjerdi, Nurettin Doğan, and Hüseyin Sayan, Gazi University.)

#### Elvan Akın-Bohner

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University of Missouri–Rolla, Department of Mathematics and Statistics, Rolla, MO 65409-0020, USA

Asymptotic Behavior of Solutions of Quasilinear Dynamic Equations

We consider a quasilinear dynamic equation which can be reduced to a half-linear equation, an Emden–Fowler equation, or a Sturm–Liouville equation. Any nontrivial solution of the quasilinear dynamic equation is eventually monotone. In other words, it can be either positive decreasing (negative increasing) or positive increasing (negative decreasing). We classify solutions of the quasilinear dynamic equations by certain integral equations.

#### Douglas Anderson

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Concordia College, Department of Mathematics and Computer Science, Moorhead, MN 56562, USA

Time Scale Integral Inequalities

Steffensen's integral inequality is shown to hold not just on the continuous calculus, but also for the difference calculus, quantum calculus, and indeed for any time-scale calculus. Steffensen's inequality on general time scales is then applied to Taylor's theorem to obtain inequalities for Taylor polynomials and Taylor remainders, again for any time scale, with the continuous, discrete, quantum and other calculuses as direct corollaries. Some other recent and classical integral inequalities are extended to the general time-scale calculus as well, including inequalities of Iyengar, Chebyshev, and Hermite-Hadamard type.

#### Szilárd András

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Babeş–Bolyai University, Department of Applied Mathematics, Cluj–Napoca, M. Kogălniceanu, No. 1, Romania

Picard Operators and Time Scales

The aim of this talk is to present some results concerning the application of the Picard operator's theory (see I. A. Rus: Picard operators and applications, Scientiae Mathematicae Japonicae, 58(2003):1, 191–219) in connection with the time scales. This theory can be used very efficiently in studying the data dependence and differentiability of solutions for integral equations and in proving Gronwall type inequalities. In this way we can prove existence and uniqueness results for integral equations (so for dynamic equations too) and we obtain also some Gronwall type inequalities in a very natural way.

# Özlem Batıt

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Ege Üniversitesi, Fen Fakültesi, Matematik Bölümü, TR-35100 İzmir, Turkey Some Function Spaces and its Duals on Time Scales

In this talk we give properties of some function spaces and define dual spaces on time scales.

# Sigrun Bodine

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University of Puget Sound, Department of Mathematics and Computer Science, Tacoma, WA 98406, USA

Asymptotic Behavior of Perturbed Dynamic Equations on Time Scales Revisited

In a well-known paper, Bohner and Lutz gave results in 2001 on the asymptotic behavior of linear systems on time scales of the form

$$x^{\Delta} = \left[ \Lambda(t) + R(t) \right] x,$$

where  $\Lambda(t)$  is a diagonal matrix satisfying an appropriate dichotomy condition and R(t) is a suitably small perturbation.

In this talk, we give new results on the asymptotic behavior of solutions of such linear dynamic systems. In particular, we will present an "averaged" Hartman–Wintner Theorem on time scales. We will also present results concerned with estimates of error terms naturally arising in the process of determining the asymptotic behavior. (Joint work with Donald Lutz, San Diego State University.)

#### Martin Bohner

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University of Missouri–Rolla, Department of Mathematics and Statistics, Rolla, MO 65409-0020, USA

A Survey of Dynamic Equations on Time Scales

We present some basic facts concerning dynamic equations on time scales and initial value problems involving them. We introduce the exponential function on a general time scale and use it to solve some dynamic equations such as first and second order linear equations, Euler—Cauchy equations, logistic equations, and Riccati equations. We also present a unification of the Laplace and Z-transform, which serves to solve any higher order linear dynamic equations with constant coefficients.

Throughout the talk, many examples of time scales will be offered. Among others, we will discuss the following examples: The two standard examples (the reals and the integers), the set of all integer multiples of a positive number (this time scale is interesting for numerical purposes), the set of all integer powers of a number bigger than one (this time scale gives rise to so-called q-difference equations), the union of closed intervals (this time scale is interesting in population dynamics; for example, it can model insect populations that are continuous while in season, die out in say winter, while their eggs are incubating or dormant, and then hatch in a new season, giving rise to a nonoverlapping population).

#### José Castillo

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San Diego State University, Department of Mathematics, San Diego, CA 92182-7720, USA

Mimetic Discretizations of Continuum Mechanics

Problems in continuum mechanics are commonly described by initial boundary value problems for a system of partial differential equations. Such problems can be discretized using finite difference, finite element, spectral, or many related techniques. Mimetic methods follow a different route: they are not used to discretize particular systems of equations, but rather to discretize the continuum theory. Vector calculus provides a powerful invariant (coordinate-free) description of continuum mechanics as does the theory of differential forms. In the vector calculus case, the operators

gradient, curl and divergence play a central role: the equation of continuum mechanics can be written in terms of these operators along with the time derivative. Thus mimetic methods for vector calculus provide discretizations of the gradient, curl and divergence, and then these discretizations are used to discretize the partial differential equations that appear in continuum mechanics problems.

The ideas of having a mimetic discretization is that if one can prove something about a particular continuum mechanics problem, for example the conservation of energy, then one should be able to prove the same thing in the discrete case. A technique to construct mimetic discretizations will be presented.

# Jan Čermák

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Brno University of Technology, Institute of Mathematics, Technická 2, CZ-61669 Brno, Czech Republic

Asymptotic Properties of First-order Delay Dynamic Equations

We present some asymptotic results for linear delay dynamic equations on time scales. We show, among others, that it is possible to generalize or extend certain results of the asymptotic theory for delay differential and difference equations.

#### Josef Diblík

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Brno University of Technology, Institute of Mathematics, Technická 2, CZ-61669 Brno, Czech Republic

Compulsory Asymptotic Behavior of Solutions of Difference Equations

In this contribution we connect retract type technique and Liapunov type approach to investigate the asymptotic behavior of solutions of system of difference equations. We show that under appropriate conditions containing conditions typical for the retract technique approach and conditions typical for the Liapunov type approach, there exists at least one solution of the system considered the graph of which stays in a prescribed domain.

#### Zuzana Došlá

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Department of Mathematics, Masaryk University, Janačkovo nám. 2a, CZ-662 95 Brno, Czech Republic

Similarities and Discrepancies between Half-linear and Linear Differential Equations

We present some recent results for the half-linear differential equation

(1) 
$$(a(t)|x'|^{\alpha}\operatorname{sgn} x')' + b(t)|x|^{\alpha}\operatorname{sgn} x = 0$$

where a, b are positive functions and  $\alpha > 0$ .

In the linear case it is well-known that the principal solution is the "smallest one" in a neighbourhood of infinity; we show that this property remains to hold for (1). (Joint work with M. Cecchi and M. Marini, University of Florence.)

We also give an extension of the Fubini theorem, which enables us to classify integral conditions for (1). Using this result and the limit characterization of the principal solution we study the coexistence of subdominant, intermediate and dominant solutions of (1). Such results are interesting in comparison with the linear equation: they show discrepancies in asymptotic properties of linear and half-linear equations. An extension to the discrete case will be discussed, too. (Joint work with I. Vrkoč, Czech Academy of Sciences.)

# Ondřej Došlý

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Nabla Symplectic Dynamic Systems and Higher-order Sturm-Liouville Dynamic Equations

Let us introduce the time scale k-th order differential operators

$$D_k^{\Delta}y := \begin{cases} y^{\Delta\nabla\dots\Delta\nabla}, & k \text{ even,} \\ y^{\Delta\nabla\dots\nabla\Delta}, & k \text{ odd,} \end{cases} \qquad D_k^{\nabla}y := \begin{cases} y^{\Delta\nabla\dots\Delta\nabla}, & k \text{ even,} \\ y^{\nabla\Delta\dots\Delta\nabla}, & k \text{ odd,} \end{cases}$$

i.e.,  $D_k^{\Delta}$  always starts with  $\Delta$ -derivative while  $D_k^{\nabla}$  always ends with  $\nabla$ -derivative, and consider the higher order dynamic equation

(\*) 
$$\sum_{\nu=0}^{n} (-1)^{\nu} D_{\nu}^{\nabla} \left( r_{\nu}(t) D_{\nu}^{\Delta} y \right) = 0.$$

We will show that this equation can be investigated as a special case of a so-called nabla symplectic system, or of a (classical, delta) symplectic system depending on the type of continuity (ld- or rd-continuous) of the functions  $r_{\nu}$ .

Various modifications of (\*) will be discussed as well.

#### Saber Elaydi

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Trinity University, Department of Mathematics, San Antonio, TX 78212-7200, USA Periodic Difference Equations and the Extension of Sharkovsky's Theorem

It is well known that Sharkovsky's theorem is limited to continuous maps on the real line. However, recently, the theorem has been extended to autonomous delay difference equations. In this talk we first show how to convert a nonautonmous difference equation to a skew-product dynamical system on a certain product space. Then we use this construction to extend Sharkovsky's theorem to nonautonomous periodic difference equations on the real line. Moreover, a general global stability result will be established. Finally, we explore the possibility of extending our results to delay periodic difference equations.

# Barnabas Garay

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Budapest University of Technology, Institute of Mathematics, Muegyetem rakpart 3–9, H-1521 Budapest, Hungary

On the Well-posedness of the Initial Value Problem for Dynamic Equations

"Existence, uniqueness, and continuous dependence", i.e., forward existence, forward uniqueness, and continuous/smooth dependence on everything what the solution may depend on — the roles of the three constitutive factors of well-posedness are investigated individually and simultaneously. A direct link to the well-posedness of the initial value problem for ordinary differential equations is established by pointing out that locally, in a well-defined technical sense, the dynamics on time scales can be embedded into ordinary differential equations dynamics. Also a somewhat parallel treatment of the inverse function theorem is given. (Joint work with Stefan Hilger, Universität Eichstätt and Peter Kloeden, Universität Frankfurt.)

## Valery Gaiko

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Belarusian State University of Informatics and Radioelectronics, Minsk, Belarus

Global Bifurcation Theory and Applications of Polynomial Dynamical Systems

Two-dimensional polynomial dynamical systems are considered. The main problem of qualitative theory of such systems is Hilbert's Sixteenth Problem on the maximum number and relative position of limit cycles. There are three local bifurcations of limit cycles: 1) Andronov-Hopf bifurcation (from a singular point of center or focus type); 2) separatrix cycle bifurcation (from a homoclinic or heteroclinic orbit); 3) multiple limit cycle bifurcation (from a multiple limit cycle of even or odd multiplicity). We connect all these local bifurcations by means of the Wintner-Perko termination principle and develop a new global approach to the complete solution of the Problem in the quadratic case of polynomial systems.

Using Erugin's two-isocline method, we construct also a canonical Kukles-type cubic system with field-rotation parameters and apply it for studying limit cycle bifurcations. In particular, we consider a special case of the Kukles system which corresponds to a generalized Liénard equation and is very important for applications, classify separatrix cycles and study global bifurcations of limit cycles.

Finally, we carry out the global qualitative analysis of a cubic centrally symmetric dynamical system which can be used as a learning model of planar neural networks.

# Zekeriya Güney

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Muğla Üniversitesi, Fen Edebiyat Fakültesi, Matematik Bölümü, TR-48000 Muğla, Turkey

Delay Differential Equations on Time Scales

Some methods of solutions for some special forms of linear delay dynamic equations on time scales are studied.

#### Gusein Guseinov

guseinov@atilim.edu.tr, http://www.atilim.edu.tr/~guseinov Atılım University, TR-06836 İncek, Ankara, Turkey Some Real and Complex Analysis Problems on Time Scales

This talk consists of three parts. In the first part we will be concerned with time scale generalizations of some integrable nonlinear evolutionary equations of the Korteweg–de Vries type. In the second part we will discuss the quantum harmonic oscillator on time scales. Finally, in the third part we will give an introduction to complex variable functions on time scales.

#### Stefan Hilger

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Symmetry Aspects in Time Scales

Symmetry is one of the fundamental principles in mathematics and physics. In this talk I will present and discuss some aspects of symmetry in time scales calculus and applications. Algebraic considerations about symmetry in the Leibniz rule lead to some recent results on discrete and periodic versions of the heat and harmonic oscillator equation.

#### Roman Hilscher

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Calculus of Variations on Time Scales

We present time scales results regarding the classical treatment of the nonlinear calculus of variations problem. The space of functions employed is that of piecewise rd-continuously  $\Delta$ -differentiable functions. We derive necessary conditions for weak local optimality, namely the Euler-Lagrange equation, the transversality condition, and the accessory problem. Assuming the coercivity of the second variation, we also establish a corresponding second order sufficiency criterion. We will emphasize techniques needed for the treatment of problems with general variable endpoints, since they differ substantially from those for fixed endpoints. (Joint work with Vera Zeidan, Michigan State University.)

#### Rukiye Kara

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Food Chain Models with Specialist and Generalist Predators

We consider two models each with three components of the chain populations. In the first model we consider a situation where a prey population X is predated by individuals of population Y. This population, in turn, serves as a favourite food for individuals of population Z. The model is governed by a system of three nonlinear first-order ordinary differential equations. From the model, predator and prey both grow logistically and Z is a generalist predator.

The second model is constituted with the logistic prey X and Holling type II predators Y and Z. The model is governed again by a system of three nonlinear first-order ordinary differential equations. In this model, Z is a specialist predator which dies out exponentially fast when its favorite food is absent or is in short supply.

To study the dynamics of the above food chain models, systems of ordinary differential equations are solved numerically. The biological parameters are varied to investigate the irregularity in the models and the existence of chaos. The chaotic attractors are obtained for suitable choices of parametric values. Sensitive dependence on the initial conditions is also investigated in the two models. (Joint work with M. Can, International University of Sarajevo.)

# Billûr Kaymakçalan

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Lower Bound Estimates for Dynamic Equations on Time Scales and Some Applications

A Langenhop type inequality is given for dynamic equations on time scales. This result is further employed to obtain lower bounds for solutions of certain dynamic equations. As an application, usage of the derived Langenhop inequality in determining the oscillatory behavior of a damped second order delay dynamic equation is

illustrated. The results obtained are important in the qualitative sense. (Joint work with Ağacık Zafer, Middle East Technical University.)

#### Peter Langenius

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Monetary Policy on Time Scales

Defining a stylized dynamic equation describing price setting behavior on a generalized time scale, one obtains a flexible modelling of monetary policy in which several policy issues of immediate interest can be easily analyzed. In particular, one obtains policy rules that are robust across a variety of time scales and thereby across model specifications. Extending the analysis, new issues, such as optimal policy cycle lengths, data frequencies, central bank transparency and degrees of so called backward and/or forward looking price setting behavior, can be adressed from a unified angle.

#### Bonita A. Lawrence

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Solutions to Multi-point Boundary Value Problems on a Time Scale

An application of the Krasnosel'skiĭ–Zabreiko fixed point theorem offers us the existence of nontrivial solutions to the second-order boundary value problem

$$y^{\Delta\Delta} + f(y^{\sigma}) = 0$$

with three-point boundary conditions

$$y(0) = 0, \quad y(p) - y(\sigma(1)) = 0,$$

defined on a time scale  $\mathbb{T}$  such that  $t \in \mathbb{T} \cap [0,1]$  and 0 . The goal of this work is to expand this result to multi-point problems as well as higher-order problems.

#### **Donald Lutz**

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On Asymptotic Expansions for Solutions of Dynamic Equations on Time Scales

For differential equations and some other types of functional equations, it is known that in the analytic case, solutions can often be expressed using certain explicit functions with remainders that have asymptotic expansions as the independent variable tends to infinity. Here we consider more general systems of dynamic equations and discuss when formal solutions of such equations are asymptotic expansions for actual solutions in the above sense. Some open problems will also be presented. (Joint work with Martin Bohner, University of Missouri–Rolla.)

#### Hua Luo

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Existence of Solutions for a Two-Point Boundary Value Problem on Time Scales

Let  $f:[0,\sigma(1)]\times\mathbb{R}^2\to\mathbb{R}$  be continuous. We consider the following nonlinear two-point boundary value problem on a time scale:

$$x^{\Delta\Delta}(t) = f(t, x(t), x^{\Delta}(t)), \quad t \in [0, 1]$$
  
 $x(0) = 0, \quad x^{\Delta}(\sigma(1)) = 0.$ 

We obtain the existence of solutions without any growth restrictions on f. (Joint work with Ruyun Ma, Northwest Normal University.)

#### Maria Meiler

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On Some Lie-Algebras of Meromorphic Vectorfields

Studying the structure of Lie-Algebras of meromorphic vectorfields reveals a lot of interesting interactions between the occurring differential equations and the difference equations which essentially arise from the generalized grading of the algebras. The structure constants of the algebras are calculated through the use of the famous complex nonlinear differential equations for the Weierstrass  $\wp$ -function.

#### **Etibar Panahov**

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A Solution of Inverse Nodal Problems for Singular Sturm-Liouville Operators

In some recent interesting works, Hald, McLaughlin and Browne, Sleeman have taken a new approach to inverse spectral theory for the Sturm–Liouville problem

$$-y'' + q(x) = \lambda y, \quad 0 \le x \le 1$$

subject to the boundary conditions

$$y'(0) - hy(0) = 0$$
,  $y'(1) + Hy(1) = 0$ 

or

$$y(0) = 0, \quad y(1) = 0.$$

The novelty of the above works lies in the use of nodal points as the given spectral data.

The inverse nodal problem on the Sturm–Liouville operator is the problem of finding the potential function q and boundary conditions using the nodal points. The purpose of this talk is to present an algorithm for solving the inverse nodal problem for singular differential operator on a finite interval. Hence, we find asymptotic formulas for nodal points and nodal length for the differential operators having singularity type  $\frac{l(l+1)}{x^2} + \frac{2}{x}$ ,  $(l \ge 0)$ ; at the point 0, the potential function is determined from the position of nodes.

#### Christian Pötzsche

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Slow and Fast Time Scales

Beyond various applications in physics and engineering science, singularly perturbed ODEs serve as a model for so-called "stiff" equations in numerical analysis. It is characteristic for such problems that (at least) two different time scales are present.

To understand the behavior of singularly perturbed systems under numerical discretization, it is important to have a geometric theory for the corresponding discrete problems available.

In this talk we present an existence and smoothness result for invariant manifolds of dynamic equations on time scales or measure chains. These subsets of the extended state space contain the "slow" motion present in the system. Moreover, we establish a saddle-point structure around these manifolds. Such a result can be seen as a first step to obtain Fenichel's geometric theory in the setting of dynamic or difference equations. Here, our basic tools are a flexible robustness theorem for exponential dichotomies under slowly varying coefficients, and an abstract integral manifold theorem.

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Function Sequence Technique for Half-linear Dynamic Equations

We develop the method involving certain function sequences that, combined with the Riccati technique, provides a new tool for investigation of oscillatory properties of half-linear dynamic equations on time scales. As applications, we give various oscillation and nonoscillation criteria (e.g., those of Hille–Nehari type and of Willet type), and comparison theorems. Many of the results turn out to be new in the discrete case and in the linear time scale case; some of the observations are new even in the continuous case.

# Andreas Ruffing

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Basic Linear Partial Difference Equations: Examples for the Three Essential Types

The method of separation of variables is applied to three standard main types of basic partial difference equations like the basic wave equation, the basic diffusion equation and the basic Laplace equation. We consider discrete extensions of the classical equations that use Askey–Wilson divided difference operators of the first order instead of partial derivatives. Separation of variables and representation of solutions in terms of basic Fourier series give explicit solutions to these models (Joint work with Kristine Ey, Munich University of Technology and Sergei Suslov, Arizona State University.)

## Bryan Rynne

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Global Bifurcation on Time Scales

We consider the structure of the solution set of a nonlinear Sturm–Liouville boundary value problem defined on a general time-scale. Using global bifurcation theory we show that unbounded continua of non-trivial solutions bifurcate from the trivial solution at the eigenvalues of the linearization, and we show that certain nodal properties of the solutions are preserved along these continua. These results extend the well-known results of Rabinowitz for the case of Sturm-Liouville ordinary differential equations. Despite this, there are various technical points in which the time-scale theory differs from the standard Sturm–Liouville theory (that is, requires some additional work). We will attempt to highlight these additional time-scale technicalities.

# Yeter Şahiner

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Oscillation of a First-order Delay Dynamic Equation on Time Scales

In this talk, we present some new sufficient conditions for a first-order order delay dynamic equation to be oscillatory.

#### Panayiotis Siafarikas

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A New Discretization Technique for the Solution of ODEs

A new analytic technique for the discretization of initial value problems of (system of) nonlinear ordinary differential equations is presented based on a functional analytic approach. By use of this method the (system of) ordinary differential equation(s) is equivalently transformed into a (system of) difference equation(s), through an operator equation (or system of operator equations). One advantage of this method is that the solution of the obtained equivalent (system of) difference equation(s), always converges to the exact solution of the (system of) ordinary differential equation(s). The obvious advantage of this method is that it is very accurate and the only errors involved are the round-off errors. Moreover, this method does not depend on the grid used and the obtained solution is proved to be unique. As an application, the presented method is applied to the Duffing oscillator equation and the Lorenz system. The results are compared with numerical ones obtained using the fourth order Runge–Kutta method. (Joint work with Eugenia Petropoulou and Efstratios Tzirtzilakis, University of Patras.)

### Burcu Silindir

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Bilkent University, Department of Mathematics, TR-06800 Bilkent, Ankara, Turkey Some Applications of Gel'fand-Dikii Formalism on Time Scales

In order to produce integrable equations, Gel'fand–Dikii formalism is one of the effective and famous methods. We have dealt with the shift Lax operators on regular discrete time scales since it is convenient with the Gel'fand–Dikii formalism.

In this talk, we briefly explain how the Gel'fand–Dikii formalism works and present some applications of the Gel'fand–Dikii formalism on time scales. As illustration, we give two field equations, four field systems, KP hierarchy, Frenkel's KdV, and the usual KdV on time scales. (Joint work with Metin Gürses, Bilkent University and Gusein Guseinov, Atılım University.)

### Petr Stehlik

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University of West Bohemia, Pilsen Univerzitni 23, CZ-31200 Plzen, Czech Republic Periodic Boundary Value Problems on Time Scales

We use the Schauder fixed-point theorem and the concept of lower and upper solutions to prove an existence of solutions for periodic BVPs on time scales, which have received recently a lot of attention. We use the monotone iterative technique to provide some of these solutions. A condition allowing the application of this method provides an interesting connection between the properties of a right-hand side function of this problem and the graininess function of a particular time scale.

### Wilhelm Sticka

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On a Picard Theorem

We extend Picard's theorem on the existence of elliptic solutions of the second kind of linear homogeneous nth-order scalar ODEs with coefficients being elliptic functions to linear homogeneous first-order systems. In particular, the qualitative Floquet-type structure of fundamental systems of solutions is determined in terms of elliptic and exponential functions, polynomials, and Weierstrass zeta functions. (Joint work with Fritz Gesztesy, University of Missouri-Columbia.)

# Christopher Tisdell

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University of New South Wales, School of Mathematics, Sydney NSW 2052, Australia Boundedness of Solutions to Dynamic Equations on Time Scales

In this talk I investigate the qualitative behaviour of solutions to the nonlinear dynamic system

(1) 
$$x^{\Delta} = f(t, x) \quad \text{for} \quad t \in \mathbb{T},$$

where t comes from a time scale  $\mathbb{T}$  and  $x^{\Delta}$  is a generalized derivative of x. The system (1) accurately describes dynamic processes where time may flow continuously

and discretely at different stages in the one model. Appropriate dynamic inequalities on f are formulated and some sufficient conditions are presented that guarantee all solutions to (1) are bounded. The general theory is then applied to give new boundedness results for discrete systems. Several examples are given that highlight the new results, including a continuous-discrete hybrid system.

# Serap Gülşan Topal

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The Quasilinearization Method for Second-order Nonlinear Dynamic Boundary Value Problems

In this talk, we present the convergence of monotone sequences for nonlinear functional boundary conditions on time scales. We construct two sequences converging to the unique solution of this problem.

# Ünal Ufuktepe

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Measure Theory on Time Scales

In this talk we study the Lebesgue delta-measure which was introduced by Professor Gusein Sh. Guseinov. We discuss the bounded convergence theorem, Fatou's lemma, the monotone convergence theorem, and the Lebesgue dominated convergence theorem on time scales. (Joint work with Ahmet Yantır and Gökşen Bacak, İzmir Institute of Technology.)

# Mehmet Ünal

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University of Bahçeşehir, Department of Mathematics and Computer Science, TR-34538 Bahçeşehir, Istanbul, Turkey

Kneser's Theorem in Quantum Calculus

While difference equations deal with discrete calculus and differential equations with continuous calculus, so-called q-difference equations are considered when studying quantum calculus. In this talk we obtain certain oscillation criteria for second-order q-difference equations, among them a quantum calculus version of the famous Kneser theorem. (Joint work with Martin Bohner, University of Missouri–Rolla.)

### Howard Warth

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University of Missouri–Rolla, Department of Mathematics and Statistics, Rolla, MO 65409-0020, USA

Oscillation Theorems for Self-adjoint Dynamic Equations

Conditions sufficient to guarantee oscillation of second-order self-adjoint dynamic equations on time scales are presented. The main result is a Kamenev type oscillation criterion. (Joint work with Martin Bohner, University of Missouri–Rolla.)

#### Nick Wintz

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Eigenvalue Comparisons for an Impulsive BVP with Sturm-Liouville Boundary Conditions

We will look at a pair of second-order differential equations that are subject to a single impulse at a given point and satisfy Sturm-Liouville boundary conditions. We will use the theory of  $u_0$ -positive operators with respect to a cone to establish the existence of the smallest eigenvalues. We also will compare these eigenvalues. (Joint work with Bonita Lawrence, Marshall University.)

#### Ahmet Yantır

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Comparisons Theorems for Self-adjoint Dynamic Equations

In this talk we establish comparison theorems for the second-order self-adjoint dynamic equation

$$L[y(t)] = [py^{\Delta}]^{\Delta}(t) + q(t)y^{\sigma}(t) = 0.$$

We interpret the zeros of the solutions of Sturm majorant of above equation by comparing the coefficients. We next give oscillation and nonoscillation criteria for this type of dynamic equations by using Riccati technique and Prüfer substitution. (Joint work with Ünal Ufuktepe, İzmir Institute of Technology.)

### Asif Yokuş

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The Exact and Numerical Solitary-wave Solutions for the Generalized Modified Boussinesq Equation

In this study, a decomposition method for approximating the solution of the generalized modified Boussinesq equation is implemented. By using this scheme, the explicit exact solution is calculated in the form of a convergent power series with easily computable components. To illustrate the application of this method, numerical results are derived by using the calculated components of the decomposition series. The obtained results are found to be in good agreement with the exact solution. (Joint work with Doğan Kaya, Fırat University.)

# Ağacık Zafer

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Oscillation and Nonoscillation of Second-order Dynamic Equations on Time Scales

We establish some oscillation and nonoscillation criteria of Wong type for second order dynamic equations and pose an open problem.

### Other Participants

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The joint paper will be presented by Ünal Ufuktepe. See page 34 for the abstract.

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The joint paper will be presented by Ömer Akın. See page 17 for the abstract.

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The joint paper will be presented by Ömer Akın. See page 17 for the abstract.

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The joint paper will be presented by Asif Yokuş. See page 36 for the abstract.

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### Social Program

### Sunday, June 26, 2005, 8 pm:

Bahçeşehir University invites you to join the Welcome Party at Governor Erol Çakır Teacher Guest House.

### Monday, June 27, 2005, 5 pm:

Sightseeing, free time. Suggestions (on participants' expenses): Visit to Dolmabahçe Palace, Ortaköy, Taksim, Çiçek Pasajı, and dinner in the Galata Tower.

### Tuesday, June 28, 2005, 5 pm:

Visit to Bahçeşehir campus with dinner, courtesy of the President (Rektör) of Bahçeşehir University.

### Wednesday, June 29, 2005, 9 am:

Istanbul tour (Topkapı Palace – Ayasofya Mosque – Archeology Museum). The Bosporus yacht tour including dinner starts at 7 pm and will take about 5 hours. Both the Istanbul tour and the Bosporus yacht tour are sponsored by the Turkish Minstry of Culture and Tourism.

#### Thursday, June 30, 2005, 8 pm:

Bahçeşehir University invites you to join the Farewell Party at Governor Erol Çakır Teacher Guest House.

#### Friday, July 1, 2005, 3:15 pm:

More sightseeing, free time (on participants' expenses).

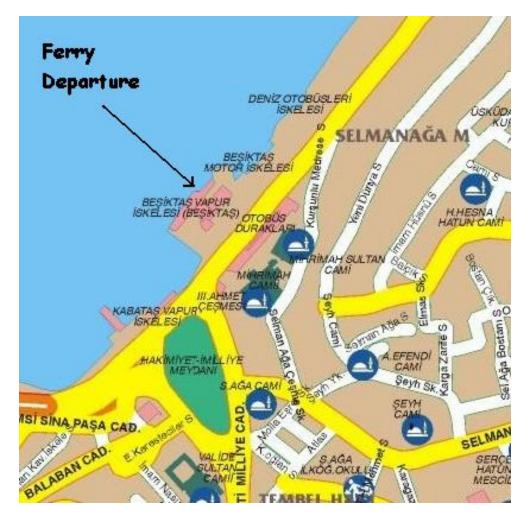
#### Information for those staying at the Gov. Erol Çakır Guest House:

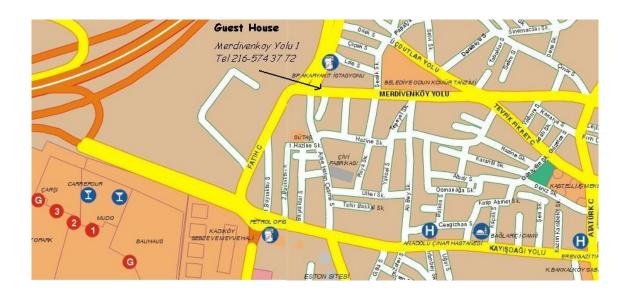
Departure from the Governor Erol Çakır Teacher Guest House is on Monday morning at 8:30 am and Tu/Th/Fr at 8:45 am by shuttle bus to Üsküdar, then by ferry to Beşiktaş (workshop site). If you miss the shuttle bus, you will have to take public transportation to the workshop site (please refer to the information sheet provided).

# Maps



MAPS 43





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#### About Istanbul

"There, God and human, nature and art are together, they have created such a perfect place that it is valuable to see." Lamartine's famous poetic line reveals his love for Istanbul, describing the embracing of two continents, with one arm reaching out to Asia and the other to Europe.

Istanbul, once known as the capital of capital cities, has many unique features. It is the only city in the world to straddle two continents, and the only one to have been a capital during two consecutive empires – Christian and Islamic. Once capital of the Ottoman Empire, Istanbul still remains the commercial, historical and cultural pulse of Turkey, and its beauty lies in its ability to embrace its contradictions. Ancient and modern, religious and secular, Asia and Europe, mystical and earthly all co-exist here.

Its variety is one of Istanbul's greatest attractions: The ancient mosques, palaces, museums and bazaars reflect its diverse history. The thriving shopping area of Taksim buzzes with life and entertainment. And the serene beauty of the Bosporus, Princes Islands and parks bring a touch of peace to the otherwise chaotic metropolis.



**Districts:** Adalar, Avcilar, Bagcilar, Bahçelievler, Bakirköy, Beşiktaş, Bayrampasa, Beykoz, Beyoğlu,

Eminönü, Eyüp, Fatih, Gaziosmanpasa, Kadıköy, Kâğithane, Kartal, Küçükçekmece, Pendik, Sarıyer, Sisli, Ümraniye, Üsküdar, Zeytinburnu, Büyükçekmece, Çatalca, Silivri, Şile, Esenler, Güngören, Maltepe, Sultanbeyli, Tuzla.



Golden Horn: This horn-shaped estuary divides European Istanbul. One of the best natural harbours in the world, it was once the centre for the Byzantine and Ottoman navies and commercial shipping interests. Today, attractive parks and promenades line the shores, a picturesque scene especially as the sun goes down over the water. At Fener and Balat, neighbourhoods midway up the Golden Horn, there are entire streets filled with old wooden houses, churches, and syn-

agogues dating from Byzantine and Ottoman times. The Orthodox Patriarchy resides at Fener and a little further up the Golden Horn at Eyüp, are some wonderful examples of Ottoman architecture. Muslim pilgrims from all over the world visit Eyüp Camii and Tomb of Eyüp, the Prophet Mohammed's standard bearer, and it is one of the holiest places in Islam. The area is still a popular burial place, and the hills above the mosque are dotted with modern gravestones interspersed with ornate Ottoman

stones. The Pierre Loti Cafe, atop the hill overlooking the shrine and the Golden Horn, is a wonderful place to enjoy the tranquility of the view.

Beyoğlu and Taksim: Beyoğlu is an interesting example of a district with European-influenced architecture, from a century before. Europe's second oldest subway, Tunel was built by the French in 1875, must be also one of the shortest offering a one-stop ride to start of Taksim. Near to Tunel is the Galata district, whose Galata Tower became a famous symbols of Istanbul, and the top of which offers a tremendous 180 degree view of the city.

From the Tunel area to Taksim square is one of the city's focal points for shopping, entertainment and urban promenading: Istiklal Cadesi is a fine example of the contrasts and compositions of Istanbul; fashion shops, bookshops, cinemas, markets, restaurants and even hand-carts selling trinkets and simit (sesame bread snack) ensure that the street is packed throughout the day un-



til late into the night. The old tramcars re-entered into service, which shuttle up and down this fascinating street, and otherwise the street is entirely pedestrianised. There are old embassy buildings, Galatasaray High School, the colourful ambience of Balık Pazarı (Fish Bazaar) and restaurants in Çiçek Pasajı (Flower Passage). Also on this street is the oldest church in the area, St. Mary's Draperis dating back to 1789, and the Franciscan Church of St. Antoine, demolished and then rebuilt in 1913.

The street ends at Taksim Square, a huge open plaza, the hub of modern Istanbul and always crowded, crowned with an imposing monument celebrating Atatürk and the War of Independence. The main terminal of the new subway is under the square, adjacent is a noisy bus terminal, and at the north end is the Atatürk Cultural Centre, one of the venues of the Istanbul Theatre Festival. Several five-star hotels are dotted around this area, like the Hyatt, Intercontinental and Hilton (the oldest of its kind in the city). North of the square is the Istanbul Military Museum.

Taksim and Beyoğlu have for centuries been the centre of nightlife, and now there are many lively bars and clubs off Istiklal Cadesi, including some of the only gay venues in the city. Beyoğlu is also the centre of the more bohemian arts scene.

Sultanahmet: Many places of tourist interest are concentrated in Sultanahmet, heart of the Imperial Centre of the Ottoman Empire. The most important places in this area, all of which are described in detail in the Places of Interest section, are Topkapı Palace, Aya Sofia, Sultan Ahmet Camii (the Blue Mosque), the Hippodrome, Kapalı Carşı (Covered Market), Yerebatan Sarnıcı and the Museum of Islamic Art.



In addition to this wonderful selection of historical and architectural sites, Sultanahmet also has a large concentration of carpet and souvenir shops, hotels and guesthouses, cafes, bars and restaurants, and travel agents.

Ortaköy: Ortaköy was a resort for the Ottoman rulers because of its attractive location on the Bosporus, and is still a popular spot for residents and visitors. The village is within a triangle of a mosque, church and synagogue, and is near

Çırağan Palace, Kabataş High School, Feriye, Princess Hotel.

The name Ortaköy reflects the university students and teachers who would gather to drink tea and discuss life, when it was just a small fishing village. These days, however, that scene has developed into a suburb with an increasing amount of expensive restaurants, bars, shops and a huge market. The fishing, however, lives on and the area is popular with local anglers, and there is now a huge waterfront tea-house which is crammed at weekends and holidays.

**Sariyer:** The first sight of Sariyer is where the Bosporus connects with the Black Sea, after the bend in the river after Tarabya. Around this area, old summer houses, embassies and fish restaurants line the river, and a narrow road which separates it from Büyükdere, continues along to the beaches of Kilyos.

Sariyer and Rumeli Kavağı are the final wharfs along the European side visited by the Bosporus boat trips. Both these districts, famous for their fish restaurants along with Anadolu Kavağı, get very crowded at weekends and holidays with Istanbul residents escaping the city.

After these points, the Bosporus is lined with tree-covered cliffs and little habitation. The Sadberk Hamm Museum, just before Sariyer, is an interesting place to visit; a collection of archaeological and ethnographic items, housed in two wooden houses. A few kilometres away is the huge Belgrade Forest, once a haunting ground of the Ottomans, and now a popular weekend retreat into the largest forest area in the city.

Üsküdar: Relatively unknown to tourists, the suburb of Üsküdar, on the Asian side of the Bosporus, is one of the most attractive suburbs. Religiously conservative in its background, it has a tranquil atmosphere and some fine examples of imperial and domestic architecture.

The Iskele, or Mihrimah Camii is opposite the main ferry pier, on a high platform with a huge covered porch in front, often occupied by older local men watching life around them. Opposite this is Yeni Valide Camii, built in 1710, and the Valide Sultan's green tomb rather like a giant birdcage. The Cinili Mosque takes its name from the beautiful tiles which decorate the interior, and was built in 1640.



Apart from places of religious interest,

Usküdar is also well known as a shopping area, with old market streets selling traditional local produce, and a good fleamarket with second hand furniture. There are plenty of good restaurants and cafes with great views of the Bosporus and the rest of the city, along the quayside. In the direction of Haydarpaşa is the lhe Karaca Ahmet Cemetery, the largest Muslim graveyard in Istanbul. The front of the Çamlıca hills lie at the ridge of area and also offer great panoramic views of the islands and river.

Kadıköy: Further south along the Bosporus towards the Sea of Marmara, Kadıköy has developed into a lively area with up-market shopping, eating and entertainment making it popular especially with wealthy locals. Once prominent in the history of Christianity, the 5th century hosted important consul meetings here, but there are few reminders of that age. It is one of the improved districts of Istanbul over the last century, and fashionable area to promenade along the waterfront in the evenings, especially around the marinas and yacht clubs.

Bağdat Caddesi is one of the most trendy and label-conscious fashion shopping streets, and for more down-to-earth goods, the Gen Azım Gündüz Caddesi is the best place for clothes, and the bit pazarı on Özelellik Sokak is good for browsing through junk. In the district of Moda is the Benadam art gallery, as well as many foreign cuisine restaurants and cafes.

Haydarpaşa: To the north of Kadıköy is Haydarpaşa, and the train station built in 1908 with Prussain-style architecture which was the first stop along the Baghdad railway. Now it is the main station going to eastbound destinations both within Turkey, and internationally. There are tombs and monuments dedicated to the English and French soldiers who lost their lives during the Crimean War (1854–56), near the military hospital. The north-west wing of the 19th Century Selimiye Barracks once housed the hospital, used by Florence Nightingale to care for soldiers, and remains to honour her memory.

**Polonezköy:** Polonezköy, although still within the city, is 25 km away from the centre and not easy to reach by public transport. Translated as village of the Poles, the village has a fascinating history: It was established in 1848 by Prince Czartorisky, leader of the Polish nationals who was granted exile in the Ottoman Empire to escape oppression in the Balkans. During his exile, he succeeded in establishing a community of Balkans, which still survives, on the plot of land sold to him by a local monastery.

Since the 1970s the village has become a popular place with local Istanbulites, who buy their pig meat there (pig being forbidden under Islamic law and therefore difficult to get elsewhere). All the Poles have since left the village, and the place is inhabited now by wealthy city people, living in the few remaining Central European style wooden houses with pretty balconies.

What attracts most visitors to Polonezköy is its vast green expanse, which was designated Istanbul's first national park, and the walks though forests with streams and wooden bridges. Because of its popularity, it gets crowded at weekends and the hotels are usually full.

Kilyos: Kilyos is the nearest beach resort to the city, on the Black Sea coast on the European side of the Bosporus. Once a Greek fishing village, it has quickly been developed as a holiday-home development, and gets very crowded in summer. Because of its ease to get there, 25 km and plenty of public transport, it is good for a day trip, and is a popular weekend getaway with plenty of hotels, and a couple of campsites.

**Şile:** A pleasant, small holiday town, Şile lies 50 km from Üsküdar on the Black Sea coast and some people even live here and commute into Istanbul. The white sandy beaches are easily accessible from the main highway, lying on the west, as well as a series of small beaches at the east end. The town itself if perched on a clifftop over looking the bay tiny island. There is an interesting French-built black-and-white striped lighthouse, and 14th century Genoese castle on the nearby island. Apart from its popular beaches, the town is also famous for its craft; Şile bezi, a white muslin fabric a little like cheesecloth, which the local women embroider and sell their products on the street, as well as all over Turkey.

The town has plenty of accommodation available, hotels, guest houses and pansiyons, although can get very crowded at weekends and holidays as it is very popular with people from Istanbul for a getaway, especially in the summer. There are small restaurants and bars in the town.

**Prince's Islands:** Also known as Istanbul Islands, there are eight within one hour from the city, in the Marmara Sea. Boats ply the islands from Sirkeci, Kabataş and Bostancı, with more services during the summer. These islands, on which monasteries were established during the Byzantine period, were a popular summer retreat for palace officials. It is still a popular escape from the city, with wealthier owning summer houses.

The largest and most popular is Büyükada (the Great Island). Large wooden mansions still remain from the 19th century when wealthy Greek and Armenian bankers built them as holiday villas. The island has always been a place predominantly inhabited by minorities, hence Islam has never had a strong presence here. Büyükada has long had a history of people coming here in exile or retreat; its most famous guest being Leon Trotsky, who stayed for four years writing 'The History of the Russian Revolution'. The monastery of St. George also played host to the granddaughter of Empress Irene, and the royal princess Zoe, in 1012. The island consists of two hills, both surmounted by monasteries, with a valley between. Motor vehicles are banned, so getting around the island can be done by graceful horse and carriage, leaving from the main square off Isa Çelebi Sokak. Bicycles can also be hired.

The southern hill, Yüle Tepe, is the quieter of the two and also home of St. George's Monastery. It consists of a series of chapels on three levels, the site of which is a building dating back to the 12th century. In Byzantine times it was used as an asylum, with iron rings on the church floors used to restrain patients. On the northern hill is the monastery Isa Tepe, a 19th century house. The entire island is lively and colourful, with many restaurants, hotels, tea houses and shops. There



are huge well-kept houses, trim gardens, and pine groves, as well as plenty of beach and picnic areas.

Smaller and less of a tourist infrastructure is Burgazada. The famous Turkish novelist, Sait Faik Abasiyanik lived here, and his house has been turned into a museum dedicated to his work, and retains a remarkable tranquil and hallowed atmosphere.



Heybeliada, 'Island of the Saddlebag', because of its shape, is loved for its natural beauty and beaches. It also has a highly prestigious and fashionable watersports club in the northwest of the island. One of its best-known landmarks is the Greek Orthodox School of Theology, with an important collection of Byzantine manuscripts. The school sits loftily on the northern hill, but permission is needed to enter, from the Greek Orthodox Patriarchate in Fener. The Deniz Harp

Okulu, the Naval High School, is on the east side of the waterfront near the jetty, which was originally the Naval War Academy set up in 1852, then a high school since 1985. Walking and cycling are popular here, plus isolated beaches as well as the public Yoruk Beach, set in a magnificent bay. There are plenty of good local restaurants and tea houses, especially along Ayyıldız Caddesi, and the atmosphere is one of a close community.

Environment: Wide beaches of Kilyos at European side of Black Sea at 25th km outside Istanbul, are attracting Istanbul residents during summer months. Belgrade Forest, inside from Black Sea, at European Side is the widest forest around Istanbul. Istanbul residents, at weekends, come here for family picnic with brazier at its shadows. 7 old water tank and some natural resources in the region compose a different atmosphere. Moğlova Aqueduct, which is constructed by Mimar Sinan during 16th century among Ottoman aqueducts, is the greatest one. 800 m long Sultan Süleyman Aqueduct, which is passing over Golf Club, and also a piece of art of Mimar Sinan is one of the longest aqueducts within Turkey.

Polonezköy, which is 25 km away from Istanbul, is founded at Asia coast during 19th century by Polish immigrants. Polonezköy, for walking in village atmosphere, travels by horse, and tasting traditional Polish meals served by relatives of initial settlers, is the resort point of Istanbul residents. Beaches, restaurants and hotels of Şile at Black Sea coast and 70 km away from Üsküdar, are turning this place into one of the most cute holiday places of Istanbul. Region which is popular in connection with tourism, is the place where famous Şile cloth is produced.

Bayramoğlu - Darica Bird Paradise and Botanic Park is a unique resort place 38 km away from Istanbul. This gargantuan park with its trekking roads, restaurants is full of bird species and plants, coming from various parts of the world.

Sweet Eskihisar fisherman borough, to whose marina can be anchored by yachtsmen after daily voyages in Marmara Sea is at south east of Istan-



bul. Turkey's 19th century famous painter, Osman Hamdi Bey's house in borough is turned into a museum. Hannibal's tomb between Eskihisar and Gebze is one of the sites around a Byzantium castle.

There are lots of Istanbul residents' summer houses in popular holiday place 65 km away from Istanbul, Silivri. This is a huge holiday place with magnificent restaurants, sports and health centers. Conference center is also attracting businessmen, who are escaping rapid tempo of urban life for "cultural tourism" and business - holiday mixed activities. Scheduled sea bus service is connecting Istanbul to Silivri.

Islands within Marmara Sea, which is adorned with nine islands, was the banishing place of the Byzantium princes. Today they are now wealthy Istanbul residents' escaping places for cool winds during summer months and 19th century smart houses. The biggest one of the islands is Büyükada. You can have a marvelous phaeton travel between pine trees or have a swim within one of the numerous bays around islands!

Other popular islands are Kınali, Sedef, Burgaz and Heybeliada. Regular ferry voyages are connecting islands to both Europe and Asia coasts. There is a rapid sea bus service from Kabataş during summers.





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