Zbl 0978.39001

Bohner, Martin; Peterson, Allan

Dynamic equations on time scales. An introduction with applications. (English)

Basel: Birkhäuser. x, 358 p. DM 144.00; öS 1051.00; sFr. 108.00 (2001).

The theory of time scales (an alternative terminology is *measure chains*) was introduced by *S. Hilger* in 1988 in his dissertation and the basic ideas of this theory are summarized in his paper published in Result. Math. 18, No. 1/2, 18-56 (1990; Zbl 0722.39001). A time scale $\mathbb{T}$ is any closed subset of the real numbers $\mathbb{R}$. For a function $f : \mathbb{T} \to \mathbb{R}$, a generalized derivative f^{Δ} is defined in such a way that it reduces to the usual derivative f' if $\mathbb{T} = \mathbb{R}$ and to the forward difference Δf if $\mathbb{T} = \mathbb{Z}$ is the set of integers. A *dynamic equation* on a time scale is an equation for an unknown function which appears in the equation along with some of its derivatives (possibly of higher order). Hence, dynamic equations are differential equations if $\mathbb{R} = \mathbb{T}$ and difference equations if $\mathbb{T} = \mathbb{Z}$.

The reviewed book is an introduction to the study of dynamic equations on time scales. Many results concerning differential equations carry over quite easily to corresponding results for difference equations, while other results seem to be completely different in nature from their continuous counterparts. The study of dynamic equations on time scales reveals such discrepancies. Moreover, since there are many time scales other than just the set of real numbers or the set of integers, the results presented in this book are much more general.

Here is a list of the chapters of the book: 1. The time scales calculus, 2. First order linear equations, 3. Second order linear equations, 4. Selfadjoint equations, 5. Linear systems and higher order equations, 6. Dynamic inequalities, 7. Linear symplectic dynamic systems, 8. Extensions.

The authors start in Chapter 1 with the fundamental results of time scale calculus where the basic properties of the generalized derivative and integral are given.

In Chapter 2 the so-called cylinder transformation is introduced and the exponential function on time scales is studied. This generalized exponential function is then used to solve the first order linear dynamic equation.

Chapters 3,4 are devoted to linear second order dynamic equations. First the attention is focused on dynamic equations with constant coefficients (both homogeneous and nonhomogeneous) and explicit formulas for their solutions using the generalized trigonometric and hyperbolic functions are presented. Wronskian determinants are introduced and Abel's theorem is used to develop a reduction of order technique to find a second solution in case one solution is already known. The time scale Laplace transformation is introduced and many of its properties are derived as well. Next, selfadjoint second order dynamic equations on time scales are studied. The classical concepts such as Green's function, Riccati equations, Prüfer transformation, oscillation, disconjugacy, eigenvalue problems and many others are extended to dynamic equations on time scales.

Chapter 5 is concerned with linear systems of dynamic equations and higher order dynamic equations. Uniqueness and existence theorems are presented, and the matrix exponential on a time scale is introduced. Among others, asymptotic properties of

solutions to linear dynamic systems are investigated.
Chapter 6 deals with dynamic inequalities on time scales. Analogues of the classical Gronwall's, Hölder's and Jensen's inequalities are presented. Two sections are devoted to Opial's and Lyapunov's inequalities.
Chapter 7 contains a brief treatment of linear symplectic dynamic systems on time scales. This is a very general class of systems that contains, for example, linear Hamiltonian dynamic systems which in turn contain Sturm-Liouville dynamic equations of higher order and selfadjoint vector dynamic equations. The last chapter deals with various extensions of the results treated in the book.
The book is self-contained and the results are presented in such a way that they are understandable to everybody who has had basic courses in calculus and linear algebra. More than 200 exercises of various degree of difficulty are presented, some of them are actually research problems in the field of time scale dynamic equations. Compared with another book devoted to dynamic time scale equations, the book of *V. Lakshmikantham, S. Sivasundaram* and *B. Kaymakcalan* [Dynamic systems on measure chains, Kluwer Academic Publishers, Dordrecht (1996; Zbl 0869.34039)], the book is more directed to linear equations and contains many results obtained in the last 5 years. This would be an excellent book to use in a topics course on dynamic equations on time scales at the advanced undergraduate level and/or beginning graduate level.

Ondřej Došlý (Brno)

Keywords : time scale; measure chain; dynamic equation; generalized derivative; cylinder transformation; time scale elementary functions; difference equations; time scale Laplace transformation; Green's function; Riccati equations; Prüfer transformation; oscillation; disconjugacy; eigenvalue problems; matrix exponential; dynamic inequalities; linear symplectic dynamic systems; Hamiltonian dynamic systems; Sturm-Liouville dynamic equations; exercises
Classification :
∗39Axx Difference equations
39-02 Research monographs (functional equations)
65Lxx Numerical methods for ODE
34-02 Research monographs (ordinary differential equations)
37J10 Symplectic mappings, fixed points