

Barić, Josipa; Bibi, Rabia; Bohner, Martin; Nosheen, Ammara; Pečarić, Josip
Jensen inequalities on time scales: theory and applications. (English) Zbl 1337.26002
 Monographs in Inequalities 9. Zagreb: Element (ISBN 978-953-197-597-1/pbk). ix, 284 p. (2015).

This book combines three areas: classical inequalities in analysis, dynamic equations on time scales and isotonic linear functionals. Classical inequalities such as the Jensen, the Hermite-Hadamard, the Hölder, the Hardy and other inequalities has a long history, while the theory of dynamic equations on time scales is rather young. The connecting point between these two areas is a conclusion that different kinds of time scales integrals are isotonic and linear functionals. Since the most classical inequalities have functional variants, a natural approach is to apply such functional inequalities to time scales setting.

The book contains ten chapters: 1. Introduction, 2. Jensen type inequalities for convex and superquadratic functions, 3. Jensen's functionals, their properties and applications, 4. Jensen's functionals for several variables, their properties and applications, 5. Improvements of the Jensen-Steffensen inequality and its converse, 6. Improvements of the Hermite-Hadamard inequality, 7. Hermite-Hadamard and Jensen-Mercer inequalities on time scales for several variables, 8. Cauchy type means and exponential and logarithmic convexity for superquadratic functions, 9. Hölder and Minkowski type inequalities and functionals, 10. Some dynamic Hardy-type inequalities with general kernels.

The first chapter contains definitions and known results about some basic objects such as convex and superquadratic functions, exponential convexity, the Jensen, the Hermite-Hadamard, the Hölder and the Hardy inequalities and gives a short introduction to time scales theory. In the second section the Jensen inequality and its converse are given together with the Hermite-Hadamard inequality, the Hölder and Minkowski inequalities with various converses and reverses of them, all for time scales integral. In the same chapter Jensen type inequalities for superquadratic functions involving time scales integrals are given. In the next two chapters, the authors consider so-called Jensen functionals on time scales in one and several variables, discuss their properties and applications, especially in connection with weighted generalized means.

Improvements of the Jensen-Steffensen inequality and its converse are given in the fifth chapter together with results on functionals which arise from those inequalities. Chapters 6 and 7 contain improvements of the Hermite-Hadamard inequalities in one and several variables case, respectively. The investigation of superquadratic functions continues in Chapter 8. In fact, the authors define functionals using Jensen's inequality, its converse and the Jensen-Mercer inequality on time scales for superquadratic functions. Several mean value theorems involving the above mentioned functionals are given, monotonicity of related Cauchy type means are proved. They also show that these functionals are exponentially convex and give some applications of them using log-convexity and exponential convexity.

Although the Minkowski inequality on time scales is already discussed in the second chapter, the ninth chapter is devoted to other variants of it and related inequalities such as the Beckenbach-Dresher, the Popoviciu, the Bellman and the Diaz-Metcalf inequalities. Finally, the aim of Chapter 10 is to extend inequalities of Hardy type with general kernels to arbitrary time scales. In this chapter, the authors give results for functions in one and several variables, interesting applications and inequalities for superquadratic functions. Also, examples of n -exponentially convex functions related to the Hardy functional are obtained.

Reviewer: [Sanja Varošanec \(Zagreb\)](#)

MSC:

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| <p>26-02 Research exposition (monographs, survey articles) pertaining to real functions</p> <p>26D15 Inequalities for sums, series and integrals</p> <p>26D20 Other analytical inequalities</p> <p>26B25 Convexity of real functions of several variables, generalizations</p> <p>26E70 Real analysis on time scales or measure chains</p> | <p>Cited in 10 Documents</p> |
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Keywords:

convex function; exponentially convex function; isotonic linear functional; Hardy-type inequalities; Hermite-Hadamard inequality; Hölder inequality; Jensen inequality; Jensen-Mercer inequality; Jensen-Steffensen inequality; mean; Minkowski inequality; superquadratic function; time scale; time scales integral