21. Determine whether or not each of the following ODEs is exact. If the ODE is exact, find its solution.

   (a) $2x + 4y + (2x - 2y)y' = 0$;

   (b) $2x + 4y + (4x - 2y)y' = 0$;

   (c) $2x\sin(y) + x^2\cos(y)y' = 0$;

   (d) $2xy^2 + (2x^2y + \sqrt{x})y' = 0$;

   (e) $e^x\sin(x) + e^y\cos(y)y' = 0$;

   (f) $2xe^y - 1 + (x^2e^y + 1)y' = 0$.

22. Find explicitly the solution of the IVP $12xy + 3 + 6x^2y' = 0$, $y(1) = 1$.

23. Determine an integrating factor for the following equations and use it to find the solution:

   (a) $4x + 3y^2 + 2xyy' = 0$ (Hint: μ is depending on x only);

   (b) $xy^2 + y - xy' = 0$ (Hint: μ is depending on y only).

24. Find an integrating factor of the form $e^{f(x,y)}$ for $M(x,y) + N(x,y)y' = 0$ in the following cases:

   (a) $\frac{M_y - N_x}{N}$ only depends on x;

   (b) $\frac{M_y - N_x}{M}$ only depends on y;

   (c) $\frac{M_y - N_x}{xM - yN}$ only depends on xy;

   (d) $\frac{M_y - N_x}{M - N}$ only depends on $x + y$.

25. Use the previous problem to find the solutions of the following problems:

   (a) $-2xy + (3x^2 - y^2)y' = 0$;

   (b) $\sin(x) - x\cos(x) - 3x^2(y - x)^2 + 3x^2(y - x)^2y' = 0$;

   (c) $3xy + 4x^2y^2 + (2x^2 + 3x^3y)y' = 0$;

   (d) $x + y - \frac{x^2}{y}y' = 0$;

   (e) $\cos(x) + (4ye^{-y} + \sin(x))y' = 0$.

26. Find an integrating factor for:

   (a) The linear first order equation $y' - f(x)y - g(x) = 0$;

   (b) The separable first order equation $y' - f(x)g(y) = 0$.

27. Read Section 2.7 and work on problems 1–4 on page 103 (Euler's method).

28. Read Section 2.8 and work on problems 3–6 on page 113 (Picard's iteration method).

29. Read Section 2.9 and work on the problems on the back of this homework assignment (difference equations).