38. Show that $y_1(t) = t+1$ and $y_2(t) = 2t+4$ solve the equation $y = ty' + (y')^2$ but that $\alpha y_1 + \beta y_2$ in general is not a solution. Why does this not contradict Theorem 3.5 as presented in the lecture?

39. Find two solutions of the equation $t^2y'' - 2ty' + 2y = 0$ such that their Wronskian is not zero (hint: try t^α). Calculate this Wronskian and give the interval where the solution is valid. Finally, find the solution of the equation that satisfies $y(1) = 3$ and $y'(1) = 4$.

40. Consider the problem $t^2y'' + 3ty' + y = 0$.
    (a) For which interval can we ensure the existence of a solution?
    (b) Find a solution y_1 of the form $y_1(t) = t^\alpha$ for some real number α.
    (c) To find another solution, try $y_2(t) = v(t)y_1(t)$ for some function v.
    (d) Make sure that the Wronskian of y_1 and y_2 is not zero (if it is zero, try (a) and (b) again). Find this Wronskian.
    (e) Now find the solution that satisfies $y(\mathrm{e}) = \frac{\mathrm{e}+2}{\mathrm{e}}$ and $y'(\mathrm{e}) = \frac{\mathrm{e}-2}{\mathrm{e}^2}$.

41. Use steps similar as in the previous problem to solve $2t^2y'' + 3ty' - y = 0$, $y(1) = 3$, $y'(1) = 0$.

42. Find the general solutions of the following equations:
    (a) $y'' - 2y' + 2y = 0$;
    (b) $y'' + 6y' + 13y = 0$;
    (c) $y'' + 2y' + 2y = 0$;
    (d) $4y'' + 9y = 0$;
    (e) $y'' + y' + y = 0$;
    (f) $y'' + 4y' + 6.25y = 0$.

43. For each of the following initial value problems, find the solution.
    (a) $y'' + 4y = 0$, $y(0) = 0$, $y'(0) = 1$;
    (b) $y'' + 4y' + 5y = 0$, $y(0) = 1$, $y'(0) = 0$;
    (c) $y'' - 2y' + 5y = 0$, $y(\frac{\pi}{2}) = 0$, $y'(\frac{\pi}{2}) = 2$;
    (d) $y'' - 2.5y' + y = 0$, $y(0) = 0$, $y'(0) = 1$.

44. For the following equations, find one solution y_1 using the characteristic polynomial, and then try to find a second solution by trying $y_2(t) = v(t)y_1(t)$ for some function v that needs to be determined. Make sure that the Wronskian of y_1 and y_2 is not zero. Then find the solution y with $y(0) = 0$ and $y'(0) = 1$.
    (a) $y'' - 2y' + y = 0$;
    (b) $y'' - 4y' + 4y = 0$;
    (c) $y'' - 6y' + 9y = 0$.