Mathematics 204   Fall 2010   Exam III

1.[20] Find the general solution of the system $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}\mathbf{x}$.

2.[20] Find the solution of the initial value problem.
$$x_1' = x_1 - x_2, \quad x_1(0) = 1$$
$$x_2' = x_1 + x_2, \quad x_2(0) = 2$$

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The next problem involves a nonhomogeneous linear system (Sec. 7.9). You are not responsible for this material on Exam III during the fall semester of 2011.

3.[20] Given that $\boldsymbol{\Psi}(t) = \begin{pmatrix} 3e^{t} & e^{-t} \\ e^{t} & e^{-t} \end{pmatrix}$ is a fundamental matrix for $\mathbf{x}' = \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix}\mathbf{x}$, find the general solution of the nonhomogeneous system $\mathbf{x}' = \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 3 \\ 1 \end{pmatrix}e^{t}$.
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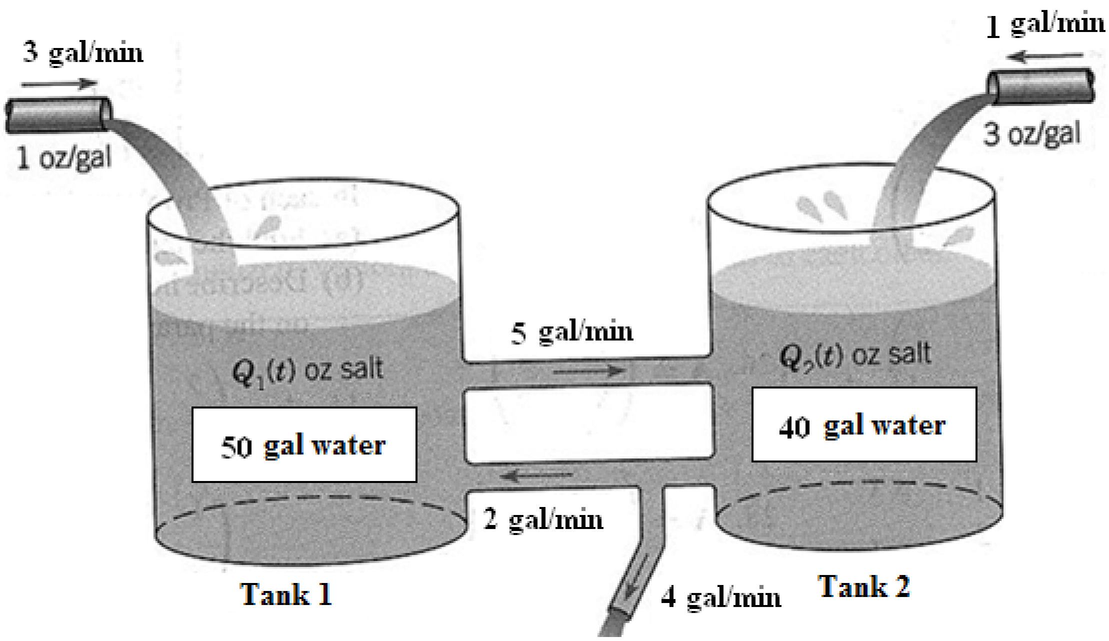
4.[20] Consider two interconnected tanks as shown below. Initially 40 oz of salt is in tank 1 and 50 oz of salt is in tank 2. Let $Q_1(t)$ and $Q_2(t)$, respectively, be the amount of salt, in oz, in each tank at time t. Assume the solution in the tanks is fully mixed. Write down, **BUT DO NOT SOLVE**, a system of differential equations and initial conditions that model the flow process.



5.[20] Find the solution of the initial value problem $y'' + 2y' + 2y = \delta(t-5)$, $y(0) = 0$, $y'(0) = 0$. Then compute the value of the solution, accurate to five decimal places, when $t = 6$.

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1.[20] Solve the initial value problem $y'' - 4y' + 5y = \delta(t - 2\pi)$, $y(0) = 0$, $y'(0) = 0$. Please express your final answer without any unit step functions.

2.[20] Solve the integro-differential equation $y'(t) = 1 - \int_0^t e^{-2\tau} y(t-\tau)\,d\tau$ subject to the initial condition $y(0) = 1$.

3.[20] Find the solution of the system

$$\frac{dx}{dt} = x + 6y$$
$$\frac{dy}{dt} = x - 4y$$

that satisfies $x(0) = 5$, $y(0) = 9$.

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The next two problems involve repeated real eigenvalues (Sec. 7.8) and nonhomogeneous linear systems (Sec. 7.9), respectively. You are not responsible for this material on Exam III during the fall semester of 2011.

4.[19] Find the general solution of the system $\mathbf{X}' = \begin{pmatrix} -6 & 5 \\ -5 & 4 \end{pmatrix}\mathbf{X}$.

5.[20] (a) If $\Phi(t)$ is a fundamental matrix for $\mathbf{X}' = \mathbf{AX}$, what is the general solution of $\mathbf{X}' = \mathbf{AX} + \mathbf{F}(t)$?

(b) Given that $\Phi(t) = \begin{pmatrix} \cos(t) & -\sin(t) \\ \sin(t) & \cos(t) \end{pmatrix}$ is a fundamental matrix for the homogeneous system $\mathbf{X}' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\mathbf{X}$, find the general solution of the nonhomogeneous system $\mathbf{X}' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\mathbf{X} + \begin{pmatrix} \sec(t) \\ 0 \end{pmatrix}$.

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4.[20] Solve the initial value problem $\mathbf{X}' = \begin{bmatrix} 1 & -8 \\ 1 & -3 \end{bmatrix}\mathbf{X}$, $\mathbf{X}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and sketch the trajectory in the phase plane.

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3.[20] Consider the matrix $\mathbf{A} = \begin{bmatrix} -1 & 2 \\ -5 & 1 \end{bmatrix}$.

(a) Determine whether $\mathbf{A}$ is singular or nonsingular. If it is nonsingular, compute $\mathbf{A}^{-1}$.

(b) Find the eigenvalues and eigenvectors of $\mathbf{A}$.