40. Read Sections 5.1 and 5.2 of the textbook.
41. The population of Utopia increases 5 percent per year. In 2000 the population was 10000. What was the population in 1970?
42. Assume that a person invests \$3000 at 12% compounded annually. Let A_n be the amount of money at the end of n years.
    (a) Find A_1, A_2, A_3, and a recurrence relation that relates A_{n+1} to A_n for $n \in \mathbb{N}$.
    (b) Find A_n for all $n \in \mathbb{N}$.
    (c) How long will it take to double the initial investment?
43. Let Ackermann's Function A be defined by $A(m,0) = A(m-1,1)\ \forall m \in \mathbb{N}$, $A(m,n) = A(m-1, A(m,n-1))\ \forall m,n \in \mathbb{N}$, and $A(0,n) = n+1\ \forall n \in \mathbb{N}$.
    (a) Compute $A(1,1)$, $A(2,2)$, and $A(2,3)$.
    (b) Use induction to show $A(1,n) = n+2$ and $A(2,n) = 3+2n$ for all $n \in \mathbb{N}_0$.
    (c) Guess a formula for $A(3,n)$ and prove it by induction.
44. Find the solutions of the following initial value problems:
    (a) $a_n = 2na_{n-1}\ \forall n \in \mathbb{N},\ a_0 = 1;$
    (b) $a_{n+1} = 7a_n - 10a_{n-1}\ \forall n \in \mathbb{N},\ a_0 = 5,\ a_1 = 16;$
    (c) $L_{n+2} = L_{n+1} + L_n\ \forall n \in \mathbb{N},\ L_1 = 1,\ L_2 = 3;$
    (d) $\sqrt{a_{n+1}} = \sqrt{a_n} + 2\sqrt{a_{n-1}}\ \forall n \in \mathbb{N},\ a_0 = a_1 = 1;$
    (e) $a_{n+1} = \sqrt{\frac{a_{n-1}}{a_n}}\ \forall n \in \mathbb{N},\ a_0 = 8,\ a_1 = \frac{1}{2\sqrt{2}}.$
45. Find all solutions of the the following recurrence relations:
    (a) $a_n = 5a_{n-1}\ \forall n \in \mathbb{N};$
    (b) $a_{n+1} = 2a_n + 8a_{n-1}\ \forall n \in \mathbb{N};$
    (c) $u_{n+2} = 7u_{n+1} - 16u_n + 12u_{n-1}\ \forall n \in \mathbb{N}.$