

51. For the following equations, find one particular solution (hint: Try ae^{bt} or $a \sin(bt) + c \cos(dt)$).
- (a) $y'' - 2y' - 3y = 3e^{2t}$;
 - (b) $y'' + 2y' + 4y = 2e^{-t}$;
 - (c) $y'' + 2y' + 5y = 3 \sin(2t)$;
 - (d) $y'' + y = 3 \sin(3t) + 4 \cos(3t)$.
52. Use the variation of parameters technique to find one particular solution:
- (a) $y'' + y' - 2y = 2t$;
 - (b) $y'' + 4y = 3 \sin(2t)$;
 - (c) $y'' + 2y' + y = 3e^{-t}$;
 - (d) $y'' + y = \tan(t)$;
 - (e) $y'' + 4y' + 4y = t^{-2}e^{-2t}$.
53. A mass weighing 2 lb stretches a spring 6 inches. At time 0 the mass is released from a point 8 inches below the equilibrium position with upward velocity of $\frac{4}{3}$ ft/sec.
- (a) Determine the function $x(t)$ which describes the subsequent free motion of the mass (ignoring any damping forces). Carefully plot x .
 - (b) Express $x(t)$ in the form $r \sin(\omega t + \theta)$.
 - (c) Calculate the times when the mass is at the equilibrium position. If the mass is at the equilibrium position, how long does it take to be there next time? How many complete vibrations undergoes the mass per second? Calculate the times when maximum displacement of the mass from the equilibrium position occurs. How long is this maximum displacement?
 - (d) Find the period, frequency, and amplitude of the motion.
54. A mass weighing 16 lb is attached to a 5 ft long spring. At equilibrium the spring measures 8.2 ft. Suppose the weight is pushed up and released from rest at a point 2 ft above the equilibrium position. Assume that the surrounding medium offers a resistance numerically equal to q times the instantaneous velocity.
- (a) For $q = 1$, work on the same questions as in the previous problem.
 - (b) For which q is the system underdamped, critically damped, and overdamped, respectively? Take one particular q in each of these cases and carefully plot the corresponding solutions.
55. Suppose that a mass is attached to a spring and that an external force of the form $F_0 \cos(\gamma t)$ is acting on the system. At time 0, the spring is released from the equilibrium position.
- (a) Find the function $x(t)$ which describes the subsequent motion of the mass.
 - (b) Sketch x for several characteristic cases. When does resonance occur?