16. For three invertible matrices of your choice from Number 15, do the following: Calculate the inverse and its eigenvalues. Guess a connection between the eigenvalues of an invertible A and the eigenvalues of A^{-1} and prove it.

17. For three matrices of your choice from Number 15, do the following: Write down the characteristic polynomial. Then plug A instead of λ, i.e., for λ^2 use A^2 and so on. Compute the result in all three cases. Guess what the result in general is and try to prove it.

18. For three matrices of your choice from Number 15, do the following: Find the matrix B given by $B = 2A^2 + A - 3I$ and calculate the eigenvalues of B. Guess what the result in general is and prove it.

19. Show that eigenvectors corresponding to different eigenvalues are linearly independent.

20. Diagonalize the following matrices, if possible. If not possible, explain why it is not possible.

    (a) $\begin{pmatrix} -4 & -3 \\ 3 & 6 \end{pmatrix}$;

    (b) $\begin{pmatrix} .85 & .03 \\ .15 & .97 \end{pmatrix}$;

    (c) $\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$;

    (d) $\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}$.

21. Find the kth power of each of the matrices from the previous problem.

22. Suppose $r_0 = r_1 = 1$ and $r_{n+1} = r_n + 2r_{n-1}$ for each $n \in \mathbb{N}$. Find a formula for r_n for each $n \in \mathbb{N}$.