39. Solve the following initial value problems:

(a) $y'' - 3y' - 10y = 0$. First, $y(0) = 1$, $y'(0) = 0$. Next, $y(0) = 0$, $y'(0) = 1$;

(b) $6y'' - 5y' + y = 0$. First, $y(0) = 4$, $y'(0) = 0$. Next: $y(0) = 0$, $y'(0) = 0$;

(c) $y'' + 3y' = 0$, $y(0) = -2$, $y'(0) = 3$;

(d) $6y'' - 7y' + 2y = 0$, $y(0) = 0$, $y'(0) = 1$;

(e) $2y'' - 3y' + y = 0$, $y(0) = 2$, $y'(0) = \frac{1}{2}$;

(f) $y'' + 4y = 0$, $y(0) = 0$, $y'(0) = 1$;

(g) $y'' + 4y' + 5y = 0$, $y(0) = 1$, $y'(0) = 0$;

(h) $y'' - 2y' + 5y = 0$, $y(\frac{\pi}{2}) = 0$, $y'(\frac{\pi}{2}) = 2$;

(i) $y'' - 2.5y' + y = 0$, $y(0) = 0$, $y'(0) = 1$;

(j) $y'' - 2y' + y = 0$, $y(0) = 0$, $y'(0) = 1$;

(k) $y'' - 4y' + 4y = 0$, $y(0) = 0$, $y'(0) = 1$;

(l) $y'' - 6y' + 9y = 0$, $y(0) = 0$, $y'(0) = 1$.

40. Consider the equation $y'' = y$.

(a) Sketch the solutions c with $y(0) = 1$ and $y'(0) = 0$ and s with $y(0) = 0$ and $y'(0) = 1$.

(b) Show that $c^2(t) - s^2(t) = 1$ for all t. Also, prove that $c' = s$ and $s' = c$.

(c) Draw the arch $y(x) = -127.7c(\frac{x}{127.7}) + 757.7$. How high is it? How long is it's base?

41. Find the Wronskian of the given pair of functions:

(a) e^{-2t} and $t\mathrm{e}^{-2t}$; (b) e^{-2t} and $\frac{3}{5}\mathrm{e}^{-2t}$; (c) $\cos t$ and $\sin t$;

(d) $\cosh t$ and $\sinh t$; (e) t^n and t^m; (f) t^n and mt^n;

(g) t and $t\mathrm{e}^t$; (h) $\cos^2 t$ and $1 + \cos(2t)$.

42. If the Wronskian of y_1 and y_2 is $3\mathrm{e}^{4t}$ and if $y_1(t) = \mathrm{e}^{2t}$, find y_2.

43. Consider the second order linear equation with constant coefficients $ay'' + by' + cy = 0$.

(a) Solve the IVP consisting of the equation and the initial conditions $y(t_0) = y_0$ and $y'(t_0) = y_0'$.

(b) Calculate the Wronskian of any two solutions of the equation.

Hint: You will need to work on three cases for each (a) and (b).