44. Find the general solution of $y''' + 6y'' + y' - 34y = 0$ if it is known that $y_1(t) = e^{-4t}\cos t$ is one solution.

45. Find the general solution of the equations in # 39 (a)–(c) by using the method presented in Example 3.7 (f) from the lecture notes, i.e., rewrite the second-order equation as a system of first-order equations and find the eigenvalues and eigenvectors of the corresponding matrix.

46. Find two solutions of the equation $t^2y'' - 2ty' + 2y = 0$ such that their Wronskian is not zero (hint: try $y(t) = t^\alpha$). Calculate this Wronskian and give the interval where the solution is valid. Finally, find the solution of the equation that satisfies $y(1) = 3$ and $y'(1) = 4$.

47. Consider the problem $t^2y'' + 3ty' + y = 0$.

    (a) Find a solution y_1 of the form $y_1(t) = t^\alpha$ for some real number α.

    (b) To find another solution, try $y_2(t) = v(t)y_1(t)$ for some function v.

    (c) Make sure that the Wronskian of y_1 and y_2 is not zero (if it is zero, try (a) and (b) again). Find this Wronskian.

    (d) Now find the solution that satisfies $y(\mathrm{e}) = \frac{\mathrm{e}+2}{\mathrm{e}}$ and $y'(\mathrm{e}) = \frac{\mathrm{e}-2}{\mathrm{e}^2}$.

48. Use steps similar as in the previous problem to solve $2t^2y'' + 3ty' - y = 0$, $y(1) = 3$, $y'(1) = 0$.

49. (First order difference equations)

    (a) Let $x_0 = 1$ and double this number to obtain x_1, double it again to obtain x_2 and so on. Find a formula for x_n, $n = 0, 1, 2, \ldots$. Use it to give x_{20}.

    (b) Let $x_0 = 1$ and muliply this number by p and add f to obtain x_1, multiply it again by p and add f to obtain x_2 and so on. Find a formula for x_n, $n = 0, 1, 2, \ldots$. Use it to give x_{20}.

50. (Second order difference equations)

    (a) Let $x_0 = x_1 = 1$. Add both numbers to obtain x_2, then add x_1 and x_2 to obtain x_3 and so on. Find a formula for x_n, $n = 0, 1, 2, \ldots$ (Hint: Try $x_n = r^n$ and use similar techniques as for differential equations). Use it to give x_{20}.

    (b) Let $x_0 = 0$, $x_1 = 1$. Multiply x_1 by $\frac{5}{2}$ and subtract x_0, to obtain x_2, then multiply x_2 by $\frac{5}{2}$ and subtract x_1 to obtain x_3 and so on. Find a formula for x_n, $n = 0, 1, 2, \ldots$. Use it to give x_{20}.