Each problem is worth 21 points. Only responses entered in the allocated space (no extra space allowed) for each problem will be graded. Present only the complete solution including all explanation (without scratch work, use separate paper for this purpose) neatly. Do not remove the staples. Grades will be posted on the web tonight and turned in to the Registrar’s office tomorrow.

79. (a) Let \( P(n) \) be the statement \( \sum_{k=1}^{n} (2k) = (n + 2)(n - 1) \). Show: If \( P(m) \) is true for some \( m \in \mathbb{N} \), then \( P(m + 1) \) is true.
(b) Let \( Q(n) \) be the statement \( e^n = e^n \). Show: \( Q(n) \) is true for all \( n \in \mathbb{N} \).

80. Give a direct \( \varepsilon-N \)-verification of the convergence of \( a_n = \frac{1}{n^{1/2}} \).

81. Let \( \{a_n\} \) and \( \{b_n\} \) be real sequences. Prove or disprove the following statements.
(a) If \( \{a_n\} \) converges to 0 and if \( \{b_n\} \) is bounded, then \( \{a_n \cdot b_n\} \) converges to 0.
(b) If \( \{a_n\} \) converges and if \( \{b_n\} \) is bounded, then \( \{a_n \cdot b_n\} \) converges.

82. Consider \( a_n = \left(1 + \frac{1}{n}\right)^n \). Which two steps did we perform in class to show that the sequence \( \{a_n\} \) is convergent? (Don’t do those steps, just explain what we did.) Which major theorem did we use here? State that theorem. In the proof of that theorem, which major axiom did we use? Give the statement of that axiom.

83. Use \( \varepsilon-\delta \)-technique to show that \( f: \mathbb{R} \to \mathbb{R} \) is continuous at \( x_0 = 2 \), where \( f(x) = x^3 + x \).

84. Show that \( g: \mathbb{R} \to \mathbb{R} \) is differentiable at \( x_0 = 0 \), where \( g(x) = \begin{cases} x^2 \sin \left( \frac{1}{x} \right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \).

85. Let \( f: \mathbb{R} \to \mathbb{R} \) be defined by \( f(x) = 2e(x) + 3x^4 - x - 2 \) for all \( x \in \mathbb{R} \).
(a) Show that \( f \) has a local minimizer in \((-1, 1)\).
(b) Is there an \( \alpha \in (-1, 1) \) with \( \frac{1 + f'(\alpha)}{2} = S(1)? \)
(c) Find \( M \subset \mathbb{R} \) such that \( f: [0, 5] \to M \) is invertible.
(d) Find \( (f^{-1})'(2e) \) provided it exists.

86. (a) State both parts of the Fundamental Theorem of Calculus.
(b) Let \( s_n = \sum_{k=1}^{n} \left( \frac{c(k)}{n} \right)^k \). Use the Fundamental Theorem of Calculus to determine \( \lim_{n \to \infty} s_n \).

87. Determine (with proving it) whether \( \int_1^\infty \frac{c(t)}{\sqrt{t}} \, dt \) exists.

88. Define “uniform convergence” and state the three main theorems about continuity, differentiability, and integrability of the limit function.