68. Determine whether the following improper integrals exist:

(a) \( \int_0^1 \frac{c(t)}{t} \, dt \);
(b) \( \int_0^\infty \frac{c(t)}{\sqrt{t}} \, dt \);
(c) \( \int_1^\infty c(t) \phi(t) \, dt \);
(d) \( \int_0^\infty s(t^2) \, dt \);
(e) \( \int_0^1 \frac{dt}{t} \);
(f) \( \int_{-1}^1 \frac{dt}{\sqrt{1-t^2}} \);
(g) \( \int_0^1 \frac{1}{t^\alpha} \, dt \);
(h) \( \int_0^\infty \frac{1}{t^\alpha} \, dt \);
(i) \( \int_1^0 \frac{1}{t^\alpha} \, dt \).

69. Let \( f_n(x) = x - \frac{x^n}{n} \). Determine the limit function \( f \) of \( \{f_n\} \) on \([0, 1]\) and decide whether \( f_n' \to f' \). Does \( \int_0^1 f_n(x) \, dx \to \int_0^1 f(x) \, dx \) hold?

70. Let \( f_n(x) = nx(1-x)^n \). Graph \( f_1, f_2, f_5, \) and \( f_{11} \) on \([0, 1]\). Determine the limit function \( f \) of \( \{f_n\} \) as \( n \to \infty \). Is the limit function continuous? Is \( f_n' \to f' \) true? How about the integral? Discuss whether \( \{f_n\} \) is uniformly convergent on \([0, 1]\).

71. Let \( f_n(x) = nx^2/(1+x^{2n}) \). What is the limit function? Show that the sequence converges uniformly on \([0, q] \) with \( q < 1 \) and also on \([\alpha, \infty) \) with \( \alpha > 1 \) but not on \( \mathbb{R} \).

72. Let \( f_n(x) = nx/(1+n^2x^2) \) for \( x \in [0, 1] \). Show that the sequence converges uniformly on \([q, 1]\) for any \( q \in (0, 1) \) but not on \([0, 1]\).

73. Show that \( \sum_{k=0}^\infty x^k(1-x) \) converges on \((-1, 1]\), but not uniformly.

74. Let \( \alpha > 1 \). Show that \( \sum_{k=1}^\infty \frac{\sin(kx)}{k^\alpha} \) is uniformly convergent on \( \mathbb{R} \).

75. Prove: If \( f_n \to f \) uniformly on \( I \) and \( g \) is bounded on \( I \), then \( f_n g \to fg \) uniformly on \( I \).

76. Prove: If \( f_n \to f \) uniformly on \( I \) and \( |f_n(x)| \geq \alpha > 0 \) for all \( n \in \mathbb{N} \) and all \( x \in I \), then \( 1/f_n \to 1/f \) uniformly on \( I \).