37. Where are the following \( f : \mathbb{R} \to \mathbb{R} \) continuous?

\[
f(x) = \begin{cases} 
  x & \text{if } x < 0 \\
  x^2 & \text{if } x \geq 0
\end{cases}
\]

and

\[
f(x) = \begin{cases} 
  0 & \text{if } x \leq 1 \\
  x & \text{if } x > 1.
\end{cases}
\]

38. Prove that there exists an \( x > 0 \) with \( \frac{1}{\sqrt{x^2+x^2}} + x^2 - 2x = 0 \).

39. For the following functions \( f \), use the Intermediate Value Theorem to show that \( f \) has a zero in the interval \( I \). Then use the Bisection Method to find an interval of length at most 0.01 that contains this zero. Then find all zeros of \( f \).

(a) \( f(x) = \frac{x^3-6x+17}{x-1}, \quad I = [2, 3] \) and \( I = [0, 2] \);

(b) \( f(x) = x^3 + \frac{5}{8} x^2 - \frac{27}{8} x + \frac{9}{8}, \quad I = [0, 1] \).

40. Determine \( \max f(M) \) provided it exists:

(a) \( f(x) = x^2 + 1, \quad M = [-4, 2] \) and \( f(x) = \sqrt{x}, \quad M = [0, 3] \);

(b) \( f(x) = -[x], \quad M = [-1, 2] \) and \( f(x) = x^2 - x[x], \quad M = [-5, 5] \).

41. Prove or disprove that \( f \) is continuous at \( x_0 \), using the “\( \varepsilon - \delta \)” Criterion:

(a) \( f(x) = x^3 + x, \quad x_0 = 2 \);

(b) \( f(x) = \frac{x^3-2}{x+1}, \quad x_0 = 1 \);

(c) \( f(x) = \begin{cases} 
  x & \text{if } x < 0 \\
  x + 1 & \text{if } x \geq 0
\end{cases}, \quad x_0 = 0 \);

(d) \( f(x) = e(x), \quad x_0 = 1 \) (see \#36).

42. Determine whether the following functions are uniformly continuous. Prove your claim.

(a) \( f(x) = 6x + 7, \quad f : \mathbb{R} \to \mathbb{R} \);

(b) \( f(x) = \frac{1}{1+x^2}, \quad f : \mathbb{R} \to \mathbb{R} \);

(c) \( f(x) = x^3, \quad f : \mathbb{R} \to \mathbb{R} \);

(d) \( f(x) = \frac{x}{x-1}, \quad f : [2, \infty) \to \mathbb{R} \).

43. Let \( a, b \in \mathbb{R} \) with \( a < b \). Find a continuous function \( f : (a, b) \to \mathbb{R} \) that is not uniformly continuous.