Exam #3, Math 309, Dr. M. Bohner, Dec 8, 2003.

Name:

The Cauchy and Lagrange mean value theorems have many applications, e.g., the result that says if $f^{(k)}(x_0) = 0$ for all $0 \leq k \leq n-1$, then, for each $x \in I \setminus \{x_0\}$ there exists a point z strictly between x and x_0 with ; the result that says that if f is three times differentiable with $f'(x_0) = 0$ and $f''(x_0) < 0$, then ; the result that says that if f is four times differentiable with $f'(x_0) = 0$, $f''(x_0) = 0$, and $f'''(x_0) \neq 0$, then ; and finally Rules that one can use to calculate the limits $\lim_{x \to 0} \frac{l(1+x)}{x} =$ and $\lim_{x \to 0,\, x>0} x l(x) =$. The nth Taylor polynomial for the function $f : I \to \mathbb{R}$ at the point x_0 is defined as $p_n(x) =$, and the Lagrange remainder theorem says that

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Now, let $a < b$ and $f : [a, b] \to \mathbb{R}$ be a function. Then $Z = (x_0, x_1, \ldots, x_n)$ is called a of $[a, b]$ provided , the gap of Z is defined by , and any $\xi = (\xi_1, \xi_2, \ldots, \xi_n)$ with is called a set of intermediate points. The Riemann sum associated with Z and ξ is defined by . Finally, f is called Riemann integrable provided

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The Fundamental Theorem of Calculus says

Part I: ;

Part II: .

As an application of the FTOC, we can show that the solution of the IVP $y' = f(x)y + g(x)$, $y(x_0) = y_0$ is given by

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Further applications contain the integration by parts rule $\int_a^b f(x)g'(x)dx =$

and the substitution formula $\int_a^b f(g(t))g'(t)dt =$. We defined locally and improperly integrable functions and showed that $\int_0^1 \frac{1}{\sqrt{x}}dx =$ and $\int_1^\infty \frac{1}{x^2}dx =$. A function $f : (a, b) \to \mathbb{R}$ is called integrable on (a, b) if $|f|$ is improperly integrable on (a, b), and it is called integrable on (a, b) if f is improperly integrable but not absolutely integrable on (a, b). If f is locally and absolutely integrable on (a, b), then it is improperly integrable on (a, b), and the inequality holds. One example of a conditionally integrable function on $[1, \infty)$ was shown in class to be . Now Chapter 7. We consider a sequence $f_n : I \to \mathbb{R}$ of functions and let $f : I \to \mathbb{R}$. We say that f_n converges to f pointwise if

and that f_n converges to f uniformly if

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and that f_n is a Cauchy sequence if

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The series $\sum_{k=0}^\infty g_k(x)$ is called uniformly convergent if is uniformly convergent. The Weierstraß M-test for uniform convergence of a series $\sum_{k=0}^\infty g_k(x)$ says that

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The sequence f_n given by $f_n(x) = x^n$ is not uniformly convergent on while it is uniformly convergent on (for the last two boxes, enter subsets of $[0, 1]$).