1. Let $A, B, C \subset X$ be sets. Prove the following:
   (a) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$;
   (b) $A \subset B \iff A^c \supset B^c$;
   (c) $A \cup B = A \iff B \subset A$;
   (d) $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$.

2. Let $f : X \to Y$ be a function. Assume $A, A_1, A_2 \subset X$ and $B, B_1, B_2 \subset Y$.
   Compare each of the following two sets (i.e., put either an “=” or “⊂” or “⊃” in between, whichever “is the best”) and prove your claim.
   (a) $f(A_1 \cup A_2)$ and $f(A_1) \cup f(A_2)$;
   (b) $f(A_1 \cap A_2)$ and $f(A_1) \cap f(A_2)$;
   (c) $f^{-1}(B_1 \cap B_2)$ and $f^{-1}(B_1) \cap f^{-1}(B_2)$;
   (d) $f^{-1}(B_1 \cup B_2)$ and $f^{-1}(B_1) \cup f^{-1}(B_2)$;
   (e) $f^{-1}(f(A))$ and $A$;
   (f) $f(f^{-1}(B))$ and $B$.

3. Work on Problem 6 of Section 1.4 in the textbook.

4. Let $f : A \to B$ and $g : B \to C$ be functions. Define a function $h : A \to C$ by $h(x) = (g \circ f)(x)$ for all $x \in A$. Prove the following:
   (a) If $f$ and $g$ both are one-to-one, then so is $h$;
   (b) If $f$ and $g$ both are onto, then so is $h$;
   (c) If $f$ and $g$ both are invertible, then so is $h$, and we have $h^{-1} = f^{-1} \circ g^{-1}$. 