26. Let $\{a_n\}$ be a real sequence. We say $\lim_{n\to\infty} a_n = \infty$ provided

$$\forall K > 0\ \exists N \in \mathbb{N}\ \forall n \geq N:\ a_n > K.$$

Prove that $\lim_{n\to\infty} \{n^3 - 4n^2 - 99n\} = \infty$.

27. Suppose $x_n \geq 0$ for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} x_n = x_0 \in \mathbb{R}$. Show $\lim_{n\to\infty} \sqrt{x_n} = \sqrt{x_0}$.

28. Prove the following statements:

(a) $\lim_{k\to\infty} a_k < c \quad \Longrightarrow \quad \exists\, N \in \mathbb{N}\, \forall\, k \geq N:\ a_k < c$;

(b) $\lim_{k\to\infty} a_k > c \quad \Longrightarrow \quad \exists\, N \in \mathbb{N}\, \forall\, k \geq N:\ a_k > c$.

29. Suppose $\{a_n\}$ is a real sequence and define $s_n = \frac{1}{n}\sum_{k=1}^{n} a_k$ for $n \in \mathbb{N}$. Prove:

(a) If $\lim_{n\to\infty} a_n = 0$, then $\lim_{n\to\infty} s_n = 0$;

(b) If $\lim_{n\to\infty} a_n = a \in \mathbb{R}$, then $\lim_{n\to\infty} s_n = a$.

30. Discuss the convergence of each of the following sequences:

(a) $\frac{n^3-6n^2+1}{2n^3+5}$, $\frac{n^2-6n+1}{n^3+5}$, $\frac{n^3-6n^2+1}{n^2+5}$, $\frac{6^n-3^n}{2\cdot 6^n+3^n}$;

(b) $\sqrt{n+1}-\sqrt{n}$, $\left(\sqrt{n+1}-\sqrt{n}\right)\sqrt{n}$, $\left(\sqrt{n+1}-\sqrt{n}\right)n$;

(c) $\prod_{k=2}^{n}\left(1-\frac{1}{k^2}\right)$, $\sum_{k=1}^{n}\frac{k^2}{n^3}$, $\sum_{k=1}^{n}\frac{1}{kn}$.

31. Discuss the monotonicity of: $8n+5$, $\frac{(-1)^n}{5n}$, $n+\frac{(-1)^n}{n}$, $\frac{1}{n^2}+\frac{(-1)^n}{3^n}$.

32. Let $a_1 = 0.1$ and $a_{n+1} = a_n(2-3a_n)$ for $n \in \mathbb{N}$. Find $\lim_{n\to\infty} a_n$.

33. Let $a_1 = 2$ and $a_{n+1} = \frac{1}{2}\left(a_n + \frac{3}{a_n}\right)$ for $n \in \mathbb{N}$. Find $\lim_{n\to\infty} a_n$.