The sequence \( \{x_n\} \) is called bounded if \( \text{Condition:} \). If \( \{x_n\} \) converges, then it also satisfies the Condition: 

If \( a_n \to \alpha \in \mathbb{R} \) and \( b_n \to \beta \in \mathbb{R} \), then \( a_n + b_n \to \), \( a_n \cdot b_n \to \), and (if \( ) \) \( \frac{a_n}{b_n} \to \) as \( n \to \infty \), and if \( a_n \leq b_n \) \( \forall n \in \mathbb{N} \), then . Next, \( \{x_n\} \) is called strictly decreasing if . The Monotone Convergence Theorem says that 

\[ a_n = \left(1 + \frac{1}{n}\right)^n \] is , since it is strictly and by . 

A sequence \( \{x_n\} \) is called a subsequence of \( \{x_n\} \) provided and using peak indices we showed that . The Theorem of Bolzano - says that if \( a \leq a_n \leq b \) for all \( n \in \mathbb{N} \), then . Next, \( g : S \to \mathbb{R} \) is called increasing if . We defined \( g \) to be continuous at \( \alpha \in S \) provided . Then we showed that this definition of continuity is equivalent to the so-called \( \varepsilon/\delta \) characterization: 

Next, a function \( g : S \to \mathbb{R} \) is called uniformly continuous if . We derived the following necessary condition for uniform continuity: 

The two major results on continuous functions are as follows:

Value Theorem:

Value Theorem:

For a limit point \( x_0 \in D \) and a function \( f : D \to \mathbb{R} \) we write \( \lim_{x \to x_0} f(x) = l \) provided . Then a function \( f : (a,b) \to \mathbb{R} \) is called differentiable at the point \( x_0 \in (a,b) \) if . The following rules hold: \( (fg)' = \) \( \left(\frac{f}{g}\right)' = \)
\[(f \circ g)' = (f^{-1})' = \]. If \(f : (a, b) \to \mathbb{R}\), then

\(x_0 \in (a, b)\) is called a local minimizer provided

and if \(f\) is also differentiable at \(x_0\), then \(x_0\) is called a critical point of \(f\) if

Now suppose \(f, g : [a, b] \to \mathbb{R}\) are differentiable on \((a, b)\) and continuous on \([a, b]\), where \(a < b\).

We proved Rolle’s theorem:

the Cauchy mean value theorem:

and the Lagrange mean value theorem:

Applications of the LMVT are the identity criterion:

and the test for a function to be strictly increasing:

Show that \(\lim_{n \to \infty} \left\{ \sum_{k=1}^{n} \frac{1}{k} - l(n) \right\}\) exists.

Is \(f : \mathbb{R} \to \mathbb{R}\) defined by \(f(x) = \begin{cases} e^{-\frac{1}{x^2}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}\) differentiable at 0?