70. State the following theorems: Bolzano–Weierstrass, Extreme Value Theorem, Lagrange Mean Value Theorem, Cauchy Mean Value Theorem.

71. Let $P(n)$ be the statement $\sum_{k=1}^{n} (2k) = (n+2)(n-1)$. Show: If $P(m)$ is true for some $m \in \mathbb{N}$, then $P(m+1)$ is true.

72. Find (and prove your claims) $\sup M$, $\max M$, $\inf M$, and $\min M$ of the set $M = \{ \frac{1}{m} + \frac{1}{n} : m, n \in \mathbb{N} \}$.

73. State the definition of convergence of a sequence and use it to prove that $\lim_{n \to \infty} \frac{1}{2n+8}$ exists.

74. State the monotone convergence theorem and use it to prove that $\lim_{n \to \infty} (1 + \frac{1}{n})^n$ exists.

75. State the $\varepsilon$-$\delta$ criterion for continuity of a function at a given point and use it to show that $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3 + x$ for all $x \in \mathbb{R}$ is continuous at $x_0 = 2$.

76. State the definition for differentiability of a function at a given point and use it to find the derivative of $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x|x|$ for all $x \in \mathbb{R}$.

77. Show that $h : \mathbb{R} \to \mathbb{R}$ defined by $h(x) = e^{\frac{x^2}{e^{1/2}}} + x^3 + x - 2$ has a zero and a local minimizer in $[0, 1]$.

78. State both parts of the fundamental theorem of calculus and use it to find $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{k}{n} e^{\left(\frac{k}{n}\right)^2}$.

79. Use integration by parts to determine whether $\int_1^2 e^{\frac{1}{x^2}} \, dx$ exists.