112. Show that $F(x) := \int_{-\infty}^{\infty} \cos(xt)e^{-\frac{t^2}{2}} dt$ exists and is continuous on $\mathbb{R}$.

113. Show that $M \subset \mathbb{R}$ is connected iff $M$ is an interval.

114. Work on Problem 1 of Section 9.3 in the textbook.

115. Work on Problem 2 of Section 9.3 in the textbook.

116. Determine whether the following arcs are rectifiable, and find the arc lengths:
   
   (a) $x(t) = \begin{pmatrix} r \cos t \\ r \sin t \\ ht^2 \end{pmatrix}$, $t \in [0, 2\pi]$ ($r, h > 0$ are fixed);
   
   (b) $x(t) = \begin{pmatrix} t \\ t \sin \left( \frac{1}{t} \right) \end{pmatrix}$, $t \in (0, 1]$, $x(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

117. Consider $x_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $x_n = \begin{pmatrix} 1 \\ n \end{pmatrix}$ for $n \in \mathbb{N}$, $y = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, and the set $M$ defined as $(\bigcup_{n \in \mathbb{N}} \overline{a_nx_n}) \cup \{y\}$, where $\overline{a_b}$ denotes the line connecting the points $a$ and $b$.
Graph $M$. Is $M$ open? Is $M$ connected? Is $M$ arcwise connected?

118. Consider two nonempty convex sets $M_1, M_2 \subset \mathbb{R}^n$. Show:
   
   (a) $M_1 - M_2 := \{x - y : x \in M_1, y \in M_2\}$ is convex.
   
   (b) If $M_1 \cap M_2 = \emptyset$, then there exist $p_0 \in \mathbb{R}^n \setminus \{0\}$ and $\alpha \in \mathbb{R}$ with
   
   $p_0^T x \geq \alpha \geq p_0^T y$ for all $x \in M_1$ and all $y \in M_2$.

119. Let $A$ be a symmetric $n \times n$-matrix and define $f : \mathbb{R}^n \rightarrow \mathbb{R}$ by $f(x) = x^T Ax$.
Show that $f$ is convex iff the matrix $A$ is positive semidefinite.

120. Determine whether the following functions $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ are convex:
   
   (a) $f(x, y) = x^2 + 3y^2 - 3xy$;
   
   (b) $f(x, y) = 2x^2 - y^2 + 4xy$.

121. Suppose $M \subset \mathbb{R}^n$ is convex. Show that a function $f : M \rightarrow \mathbb{R}$ is convex iff the so-called epigraph $\{ (x, y) \in M \times \mathbb{R} : f(x) \leq y \}$ is convex.

122. Suppose $M \subset \mathbb{R}^n$ is convex and $f : M \rightarrow \mathbb{R}$ is strictly convex. Assume that there exists $x_0 \in M$ with $f(x_0) = \max f(M)$. Prove that $x_0 \in \partial M$.

123. Suppose $M \subset \mathbb{R}^n$ is closed and convex, and let $x_0 \in \mathbb{R}^n$. Prove that there exists exactly one $a \in M$ with $d(x_0, M) = \|x_0 - a\|$.