129. A function \( f : \mathbb{R}^2 \to \mathbb{R} \) is called harmonic provided that it has second-order partial derivatives and \( \frac{\partial^2 f}{\partial x^2}(x,y) + \frac{\partial^2 f}{\partial y^2}(x,y) = 0 \) for all \((x,y) \in \mathbb{R}^2\). Which of the following functions are harmonic?

(a) \( f(x,y) = e^x \cos y \);
(b) \( g(x,y) = x^2 - y^2 \);
(c) \( h(x,y) = x^2 - y^3 \).

130. Use the definition of the derivative to find \( f' \) for the following functions:

(a) \( f(x,y,z) = \frac{4x+y-3z-3}{1+2y-z} \), \( f : \mathbb{R}^3 \to \mathbb{R}^2 \);
(b) \( f(x) = Ax - b \), where \( A \) is an \( m \times n \)-matrix and \( b \in \mathbb{R}^m \), \( f : \mathbb{R}^n \to \mathbb{R}^m \);
(c) \( f(x,y) = x^2y + y^2 + 1 \), \( f : \mathbb{R}^2 \to \mathbb{R} \);
(d) \( f(x,y) = 4x^2 + 8xy + y^2 \), \( f : \mathbb{R}^2 \to \mathbb{R} \);
(e) \( f(x) = x^T Ax \), where \( A \) is a symmetric \( n \times n \)-matrix, \( f : \mathbb{R}^n \to \mathbb{R} \);
(f) \( f(x) = x^T Ax - b^T x \), where \( A \) is a symmetric \( n \times n \)-matrix and \( b \in \mathbb{R}^n \), \( f : \mathbb{R}^n \to \mathbb{R} \).

131. State and prove a theorem that generalizes the results of #126 to an arbitrary totally differentiable function.

132. Let \( f : \mathbb{R}^2 \to \mathbb{R} \) be given by \( f(x,y) = \begin{cases} xy^2-x^2 & \text{if } x^2+y^2 > 0 \\ 0 & \text{if } x = y = 0. \end{cases} \)

Calculate the second derivative of \( f \). Is \( f \in C^2 \)?

133. Work on Problem 1 of Section 11.3 in the textbook.

134. Work on (a) Problem 5, (b) Problem 7, and (c) Problem 13 of Section 11.3.

135. Let \( I_1, I_2 \subset \mathbb{R} \) be open intervals with \( a, b \in I_1, \ a < b, \) and \( \phi : I_1 \to \mathbb{R} \) is a differentiable, strictly increasing function with \([\phi(a), \phi(b)] \subset I_2\). Prove: If \( f \in C(I_1 \times I_2) \), then
\[
\int_a^b \left\{ \int_{\phi(a)}^{\phi(b)} f(x,t) \, dt \right\} \, dx = \int_{\phi(a)}^{\phi(b)} \left\{ \int_a^b f(x,t) \, dx \right\} \, dt.
\]

136. Calculate \( \int_1^2 \left\{ \int_1^x \frac{1}{t} e^{x/t} \, dt \right\} \, dx \).

137. Show that there is no function \( f : \mathbb{R}^2 \to \mathbb{R} \) with \( f_x(x,y) = x \) and \( f_y(x,y) = xy \) for all \( x, y \in \mathbb{R} \).