85. Prove Theorem 9.2 from the lecture notes, i.e., the set \( \mathbb{R}^n \) is a vector space.

86. Prove the following statements, where \( x, y, z \in \mathbb{R}^n \).

   (a) \( \|x\| \geq 0 \) and \( \|x\| = 0 \) iff \( x = 0 \);
   (b) \( \|\lambda x\| = |\lambda|\|x\| \) for all \( \lambda \in \mathbb{R} \);
   (c) \( x \perp y \) iff \( \|x + y\|^2 = \|x\|^2 + \|y\|^2 \);
   (d) \( (x - y) \perp (x + y) \) iff \( \|x\| = \|y\| \);
   (e) \( (x - z) \perp (y - z) \) iff \( \|x - z\|^2 + \|y - z\|^2 = \|x - y\|^2 \);
   (f) \( \|x + y\|^2 + \|x - y\|^2 = 2 (\|x\|^2 + \|y\|^2) \);
   (g) \( \|x - y\| \geq |\|x\| - \|y\|| \);
   (h) \( \langle x, y \rangle = \frac{\|x + y\|^2 - \|x - y\|^2}{4} \).

87. Show that the \( \ell_1 \)-norm is a norm.

88. Show that the sup-norm is a norm.

89. Show that the space of all real sequences \( x = \{x_k\}, y = \{y_k\} \) is a metric space with \( d(x, y) = \sum_{k=1}^{\infty} \frac{|x_k - y_k|}{1 + |x_k - y_k|} \).

90. Work on Problems 1a and 2a of Section 8.2 from the textbook.