152. Work on Problem 1 (a) in Section 11.5 of the textbook.

153. Define \( f : \mathbb{R}^2 \to \mathbb{R}^2 \setminus \{(0, 0)\} \) by \( f(x, y) = (e^x \cos y, e^x \sin y) \).

(a) Is \( f \) onto?

(b) Show that \( f \) is locally one-to-one.

(c) If \( f^{-1} \) is a local inverse function of \( f \), find \( (f^{-1})' \).

(d) Is \( f \) one-to-one?

154. Does the function \( f(x, y) = \left( \begin{array}{c} \log(x+y+1) \\ 1-x+2y-y \sin x \end{array} \right) \) possess an inverse function in a neighborhood of \( (0, 0) \)? If possible, calculate \( (f^{-1})'(0, 1) \).

155. Use the Newton method with start value \( (1, 1) \) to find the solution of the problem

\[
xe^{xy} = 1 \quad \text{and} \quad ye^{xy} = 2
\]

up to five decimal places.

156. Work on Problem 2 (a) in Section 11.5 of the textbook.

157. Show that there exists \( \varepsilon > 0 \) so that for \( G = B_\varepsilon(0, 0) \) there is a function \( z \in C^1(G) \) with \( z(0, 0) = 0 \) and

\[
x^4z(x, y) + 2 \cos y \sin(z(x, y)) = 0 \quad \text{for all} \quad \begin{pmatrix} x \\ y \end{pmatrix} \in G.
\]

Also calculate the partial derivatives of \( z(x, y) \) in \( G \) (depending on \( z \)).

158. Show that there exists \( \varepsilon > 0 \) so that for \( G = B_\varepsilon(0, 0) \) there are functions \( u, v \in C^1(G) \) with \( u(0, 0) = 1, v(0, 0) = 2, \) and

\[
\begin{align*}
x^2 + y^2 - u^2(x, y) - v^2(x, y) + 5 &= 0 \\
x^2 + 2y^2 + 3u^2(x, y) + 4v^2(x, y) - 19 &= 0
\end{align*}
\]

for all \( \begin{pmatrix} x \\ y \end{pmatrix} \in G \).

Also calculate the partial derivatives of \( u \) and \( v \) in \( G \) (depending on \( u \) and \( v \)).